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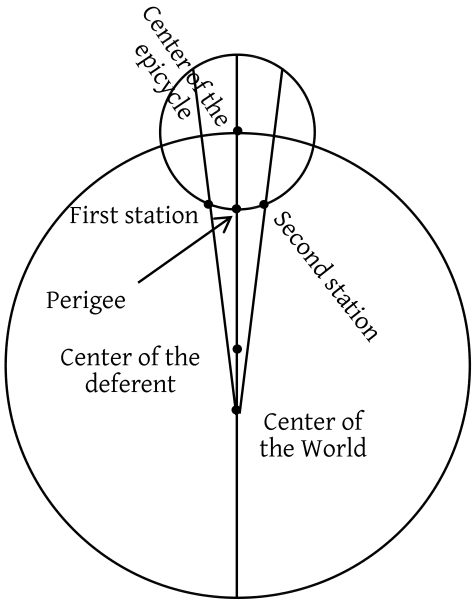


[Figure 2]

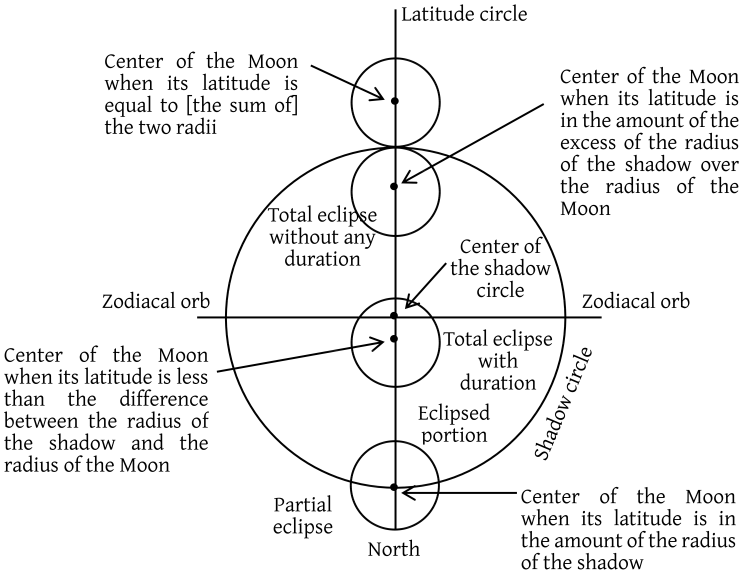
Section [Seven]

On Clarifying the Different Circumstances of Lunar and Solar Eclipses with Respect to Differences in Latitude, etc.

[1] In Chapter Thirteen, it was stated that if the lunar latitude is in the amount of the radius of the shadow [plus] the radius of the Moon, the Moon in its transit will become tangent to the shadow circle, and no lunar eclipse will occur. If the [latitude] is greater, the Moon will not become tangent; if it is less [than the shadow radius plus the lunar radius], but greater than the excess of the shadow radius over the lunar radius, there will be a partial lunar eclipse. If it is equal to that excess, there will be a total lunar eclipse without any duration. If it is less than the excess, the lunar eclipse will be total, and there will be duration.

[2] As an example, let us assume a lunar eclipse, where the shadow diameter is 84 minutes and the Moon's diameter is 32 minutes. [Then] the radius of the shadow is 42 minutes and the Moon's radius is 16

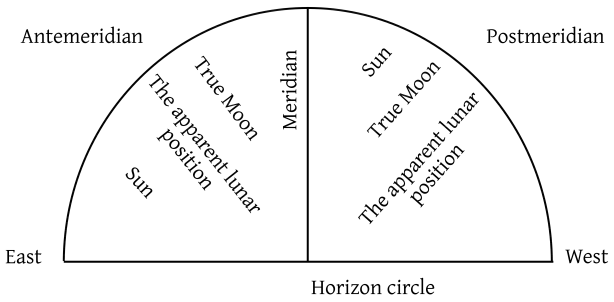
minutes; the sum of these two is 58 minutes. Therefore, if the Moon's latitude is 58 minutes, no lunar eclipse will occur, and the Moon will be externally tangent to the shadow circle. If it is greater, it will not become tangent. If the Moon's latitude is less than this, there will be a lunar eclipse. Since the excess of the shadow's radius over the Moon's radius is 26 minutes, if the Moon's latitude is greater than that, then a partial lunar eclipse will occur. If it [equals] the same amount, a [total] lunar eclipse will occur without any duration. If [the latitude] is less, there will be duration. From this illustration, this matter will easily be understood:



[Figure 1]

[3] Concerning solar eclipses: what we have stated, namely that the times of true conjunction are always closer to noontime than the [apparent] solar eclipse, is due to the fact that since the Moon moves in the sequence of the signs from west to east and the apparent position of the Moon is nearer to the horizon than the true position, then before noon the apparent Moon will reach the Sun first, then afterward the true Moon, and conversely after noon. This meaning should become

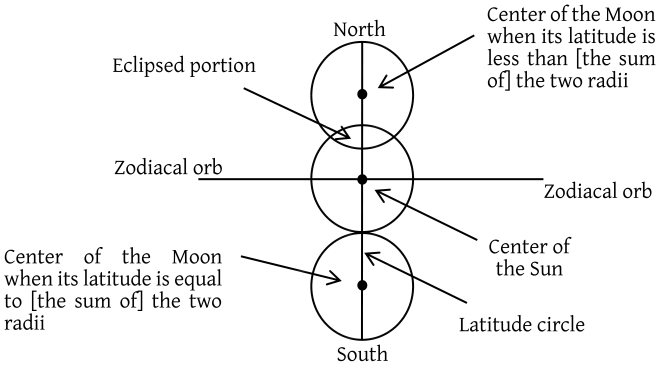
clear from this illustration:



[Figure 2]

[4] From this illustration, in the [part of the] illustration that we have drawn [for] before noon, it is clear that when the Moon is in conjunction with the Sun, [having moved] from west to the east, which conforms to the sequence of the signs, first the apparent position of the Moon will reach the Sun and a solar eclipse will occur. After that the true position will reach it, and there will be conjunction. Thus, the times of the solar eclipse will be sooner than the times of conjunction, and the conjunction will be nearer midday. And in the illustration that we have drawn for after noon, the [Moon's] true position will reach the Sun first, and a conjunction will occur; afterwards, the apparent position of the Moon will reach the Sun, and there will be a solar eclipse. Thus, the times of conjunction will be sooner than the times of solar eclipse, and conjunction will be closer to midday—God is all-knowing.

[5] Let us assume, as an example of a solar eclipse, just as we cited for the Moon, a time when the diameter of the Sun is 32 minutes, and the diameter of the Moon is 34 minutes; thus the [sum of the] two radii is 33 minutes. If the apparent latitude of the Moon is this amount, the Moon will be seen to be tangent to the Sun, and a solar eclipse will not occur. If it is less [than this amount], a solar eclipse will occur. If greater, [the Moon] will not come into tangency, as in this illustration:



[Figure 3]

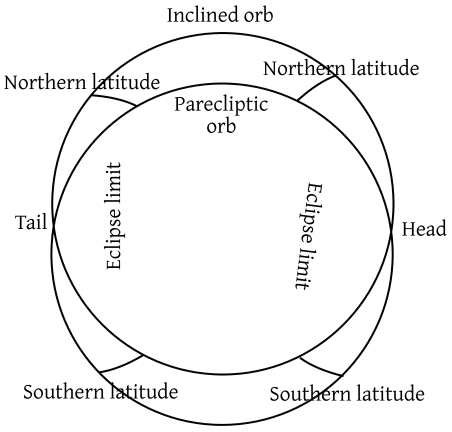
[6] To explain the times between lunar eclipses and solar eclipses, we say: since parallax is not taken into account for a lunar eclipse; and the maximum size of the shadow radius at the time when the Moon is in the epicyclic perigee is up to 46 minutes; and the maximum size of the Moon's radius is up to 18 minutes, the sum of these two amounts being 64 minutes, then when the lunar latitude is more than this amount, a lunar eclipse is not possible. This amount of latitude will occur at a distance of 12 degrees and a fraction from the node, because one degree of latitude will occur at a distance of approximately $15+\frac{1}{2}$[1] degrees. Thus, the Moon's inclined orb may be divided into four parts: in two parts of which a lunar eclipse is possible and in two parts of which a lunar eclipse is not possible. Each of those two parts [in which a lunar eclipse is possible] has a lunar eclipse limit of 24 degrees plus a fraction; each of the other two parts is 156 degrees.

[7] Since a lunar eclipse is at the time of opposition, there is no doubt that the Sun must be in the lunar eclipse limit so that the Moon can be eclipsed when in opposition to it. And since the lunar eclipse limit is no more than 25 degrees, and the Sun traverses this amount in about 25 days, and during this period the Moon is not able to make a complete revolution, hence lunar eclipses are not possible in two consecutive

1. In the revised version, "$15+\frac{1}{2}$" has been correctly changed to "$11+\frac{1}{2}$."

months. In six months, during which the Sun travels from the vicinity of one node to the vicinity of the other node, a lunar eclipse may be able to occur.

[8] If the Sun on rare occasions be at the end of an eclipse limit, for example if it has passed 10 degrees beyond the head and a lunar eclipse occurs, 5 months later the Sun will have traversed about 150 degrees, and the distance between it and the position of the head will be 160 degrees. The tail in these 5 months will have traversed about 8 degrees in the counter-sequence [of the signs]; thus, the distance between it and the Sun is no more than 12 degrees. For this reason, the Sun will have reached the beginning of the lunar eclipse limit; therefore, it will be possible for the Moon to be eclipsed once again five months after the first eclipse, but there will not be a total eclipse for either one.

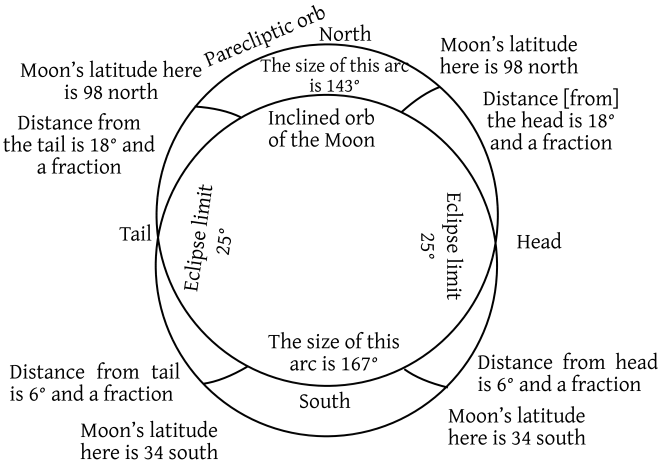


[Figure 4]

[9] But in seven months the Sun will have traversed approximately 205 degrees. If we assume that it was at the beginning of the lunar eclipse limit and the Moon was eclipsed, after seven months the Sun will have passed each of the two nodes and will be 13 degrees beyond the position of the second node. [This is] because if it traverses 12 degrees, it will reach the first node, and [then] if it traverses 180 degrees, it will reach the position of the second node; and when it completes 205

degrees, it will have traversed another 13 degrees. Thus, it has gone beyond the lunar eclipse limit. [This is] despite the fact that in this [same] period, the node has moved 11 degrees in the counter-sequence of the signs, the distance of the node from the Sun thus becoming far too much. Hence, it is not possible for two lunar eclipses to be seven months apart.

[10] As for a solar eclipse in the fourth clime, we stated in the chapter on parallax that the [lunar] parallax cannot be greater than 64 minutes. And half the two diameters, i.e., the diameters of the Sun and Moon, do not become greater than 34 minutes. When the Moon's latitude is northerly, the latitudinal parallax must be subtracted from the lunar latitude, the remainder thus being the apparent latitude. Now at 98 minutes of latitude, whenever the latitude is northerly and the parallax is at its full maximum, there will result an apparent latitude of 34 minutes. For this reason, the maximum solar eclipse limit in the north is where the latitude is 98 minutes; there, the distance of the Moon from the node is 18 degrees plus a fraction. But since for south [latitudes] the parallax has to be added to the latitude in order to get the apparent latitude, the maximum southerly latitude in which a solar eclipse is possible is 34 minutes. Where the latitude is 34 minutes, the distance from the node is $6+\frac{1}{2}$ degrees. Then the parecliptic orb may be divided into 4 parts, similar to what we stated for the Moon. These parts, however, are unequal. The two parts that are the solar eclipse limits are each 25 degrees each; of that total, $18+\frac{1}{2}$ is in the northern direction and $6+\frac{1}{2}$ is in the southern direction. The two parts in which a solar eclipse is not possible are also unequal: the northern part is 143 degrees, and the southern part is 167 degrees, as in this illustration:



[Figure 5]

[11] And since each of these two parts, which is the solar eclipse limit, is no more than 25 degrees, a solar eclipse is not possible in [each of] two consecutive conjunctions, because the Sun will have traversed about 29 degrees during this period and be outside the solar eclipse limit. Since one part in which a solar eclipse is not possible is 143 degrees, and the Sun in five months can traverse 150 degrees, then in five months two solar eclipses are easily possible. That is so because it had passed the head in the first solar eclipse, and it had not reached the tail in the second solar eclipse. Likewise, for seven months, on the assumption that in the first solar eclipse it has not reached the tail and its distance from the tail is about 18 degrees, and seven months later that it has traversed 205 degrees, i.e., the Sun is at its slow speed, it would have passed beyond the head and would have overtaken its initial position by 7 degrees if the head had no movement. However, since the head has also moved 11 degrees, the distance is 18 degrees, and the Sun will not be outside the solar eclipse limit.

[12] Because in a southern locality the same is applicable, but south [of the ecliptic], then a solar eclipse will be possible at a distance of 18 degrees from the head in the sequence [of the signs] for northern

localities, and for southern localities, at a distance of 18 degrees in the counter-sequence, also from the head. Considering this, the solar eclipse limit will be 36 degrees, and for two successive conjunctions a solar eclipse is possible but on two [different] sides of the world, i.e., north and south; in one location, however, it is not possible.

[13] This is an explanation of this matter as appropriate for this place. These values that we gave in the examples, if divergent from true ones, should be excused, inasmuch as this is not the place for correcting practical [values/calculations]. For providing an account of the configuration (*hayʾat*), this is of sufficient extent.

Section [Eight]
On Conceptualizing the Equation of Time

[1] In the Ninth Chapter of Book III, it has been stated that the difference between mean days and true days, which is expressed as the equation of time, is composed of two differences. One of these is due to fastness and slowness of the Sun on account of its uniform motion about the center of the eccentric and the consequent irregularity about the center of the World. Its greatest extent amounts to twice the maximum equation. In the apogean half, i.e., in the half of the revolution in which the apogee is at the midpoint of that half, the true days will be shorter than the mean ones because of the slower speed, while in the other half they will be longer. The second [difference] is what occurs due to the difference between degrees of the zodiacal orb and the parts on the equinoctial upon the transit of the meridian circle, or[1] rising for horizons of the equator. The maximum of this difference is 5 degrees. In the two quarters in which the two equinox points are the midpoints, i.e., from the midpoint of Aquarius to the middle of Taurus

1. Reading *yā* instead of *bā*.