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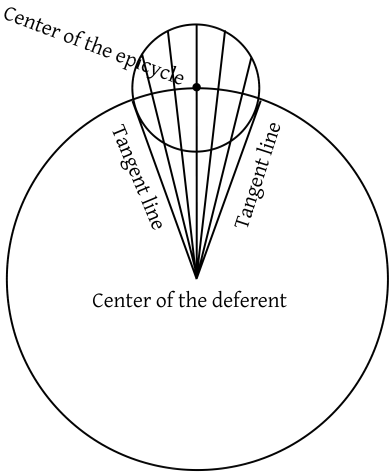
Section [Six]
On Explaining How to Determine the Stationary Positions of the Planets on the Epicycle Orb

[1] In Chapter Ten of Book I[1], we have stated that Ptolemy in the *Almagest* has shown that for a line coming from the center of the World to the epicycle orb and passing through it, if half [the line] inside the epicycle to that outside it between the center of the deferent[2] and the circumference of the epicycle has the same ratio as that of the motion of center to the proper motion of the planet, then when [the planet] reaches that line in the opposite half, [the planet] will become stationary. To explain this, let us draw an illustration of the deferent orb and epicyclic orb, conceiving lines coming from the deferent center to the epicyclic orb. Without too much thought, it is evident that the line tangent to the epicycle does not enter inside the epicycle, and the longest line falling within the epicycle is one that passes through its center, i.e., its diameter. And when a line comes from the center of the World to the epicycle center, that part of the line inside the epicycle orb will be the epicycle diameter, and that which falls between the circumference of the epicycle and the center of the deferent will be the shortest line that comes from the deferent center to the circumference of the epicycle. And of the lines that fall between the tangent line and this line, those falling nearer the epicycle center have longer internal portions and shorter external ones. And those closer to the tangent line will be the converse of this, their internal portions being shorter and their external portions longer. For any two lines assumed

1. This should be Book II.

2. In the revised version, "the center of the deferent" has been correctly changed to "the center of the World." Note that in the text of the edition, *ʿālam* should be *ḥāmil* to reflect the original version.

at equal distance on either side, these portions on both will be equal, as can be clearly seen in this illustration:



[Figure 1]

[2] And it is evident that if one makes the ratio of one quantity to another, e.g., 1 to 10, which is a tenth of it, and then if one makes a ratio from a larger quantity than the first to a smaller quantity than the second, such as 2 to 8, 2 being larger than 1 and 8 being smaller than 10, this ratio being $\frac{1}{4}$, the first ratio is necessarily smaller than the second, $\frac{1}{10}$ being smaller than $\frac{1}{4}$. Therefore, the ratio of half the internal portion of the line closer to the tangent point to the external portion is smaller than the ratio of the internal portion to the external portion of another line farther from that point. And of all ratios, the largest is the ratio of the radius of the epicycle to that line that lies between the center of the deferent and the circumference of the epicycle, since its internal portion is the largest of all internal portions and its external portion is the smallest of all the external portions. And if we consider these lines coming from the center of the World, it will make no difference in the ratios.

[3] Now that these preliminaries have been made clear, we say that any epicycle whose ratio of its radius to the line falling between it and

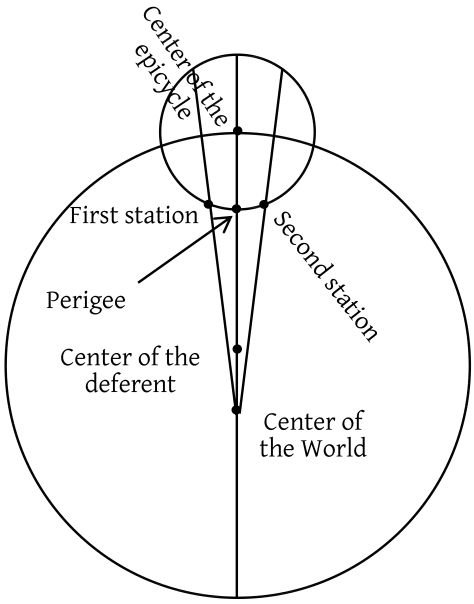
the center of the World is less than the ratio of its motion of center to the motion of the revolving planet, for that epicycle no line can be assumed such that the ratio of half its internal [portion] to its external portion be that ratio, because all ratios are less than the first ratio and the ratio of the first is less than the ratio of the motion of center to the proper motion. Accordingly, all ratios will be less than the ratio of the motion of center to the proper motion. An example of this is the lunar epicycle. When it is at the nearest distance, its radius is found to be $7+\frac{2}{3}$ parts, and from the Moon's nearest distance to the Earth is approximately 33 parts. When we subtract $7+\frac{2}{3}$ parts from this amount, there remain $25+\frac{1}{3}$ parts; this is the length of a line from the center of the World reaching the epicycle. The ratio of the epicycle radius to this amount is approximately $\frac{3}{10}$. The motion of its center is 13 degrees, 11 minutes, and the proper motion is 13 degrees, 4 minutes. The ratio of one to the other is approximately a ratio of equality, and $\frac{3}{10}$ is less than this ratio. Thus, in the lunar epicycle no [such] line can be assumed for this ratio; for this reason, there would be neither station nor retrogression for the Moon.

[4] If we assume that the ratio of the radius of an epicycle orb to the external line is like the ratio of motion to motion in all circumstances, the other ratios, which are less than the ratio of the epicycle radius, are less than the ratio of motion to motion. Therefore, the planet will become stationary at the perigee point of that epicycle and then proceed again with direct motion; for this epicycle there will be no retrogradation. When the ratio of the epicycle radius to the external line is greater than the ratio of the motion, [then] on either side of the radius are found two lines [each of] whose ratios are equal to the ratio of the motion[s]. It follows that every given line between those two lines will have a ratio greater than the ratio of the motion[s], inasmuch as they are closer to the radius. Every line falling outside these two lines on account of being closer to the tangent point will have a ratio smaller

than the ratio of the motion[s]. Therefore, as long as the planet has not reached one of these two lines, it will have direct motion. When it reaches the first line, [the planet] will become stationary, since the ratio of that line is equal to the ratio of the motion. It will then cross those two lines.[1] As long as it has not yet reached the second line on the other side of the perigee, it will retrograde. When it reaches the second line, the [planet] will become stationary. When it passes it and the ratio becomes less [than the ratio of the motions], it will undergo direct motion.

[5] As an example, for the planet Saturn let us assume the epicycle center to be at the apogee of the deferent. Since we take the radius of the deferent to be 60, the eccentricity 3 parts, 25 minutes, [the distance] from the epicycle center to the center of the World 63 parts, 25 minutes, and the radius of the epicycle $6+\frac{1}{2}$ parts, then between the center of the World and the epicycle orb is about 57 parts. The ratio of the [epicycle] radius to this quantity is approximately a ratio of $\frac{1}{9}$. The center moves 2 minutes every day, while the proper motion is 57 minutes. The ratio of 2 to 57 is approximately $\frac{1}{3}$ of $\frac{1}{9}$. Thus, the ratio of line to line is much greater than the ratio of motion to motion. This being the case, two lines will fall on the two sides of the center, one passing through the first stationary point and the other passing through the second stationary point; this is because the ratio of the internal portion of those two lines to the external portion is equal to the ratio of [one] motion to the [other] motion. Between those two lines, the planet will retrograde.

1. In the revised version, “those two lines” have been correctly changed to “that line.”



[Figure 2]

Section [Seven]

On Clarifying the Different Circumstances of Lunar and Solar Eclipses with Respect to Differences in Latitude, etc.

[1] In Chapter Thirteen, it was stated that if the lunar latitude is in the amount of the radius of the shadow [plus] the radius of the Moon, the Moon in its transit will become tangent to the shadow circle, and no lunar eclipse will occur. If the [latitude] is greater, the Moon will not become tangent; if it is less [than the shadow radius plus the lunar radius], but greater than the excess of the shadow radius over the lunar radius, there will be a partial lunar eclipse. If it is equal to that excess, there will be a total lunar eclipse without any duration. If it is less than the excess, the lunar eclipse will be total, and there will be duration.

[2] As an example, let us assume a lunar eclipse, where the shadow diameter is 84 minutes and the Moon's diameter is 32 minutes. [Then] the radius of the shadow is 42 minutes and the Moon's radius is 16