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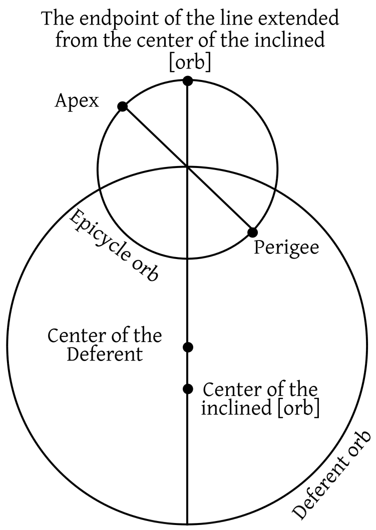
Section [Five][1]

On the Configuration of the Epicycle Orbs of the Planets According to the Theory of Abū ʿAlī ibn al-Haytham

[1] This man was a prominent mathematician, and the configuration of the orbs as solid bodies is mostly taken from his statements. He has a treatise explaining the orbs of the planets' epicycles in such a way that the various motions result from them. He states that each of the upper planets has three epicycle orbs, one enclosing another.

[2] The first orb, which is inside the two other orbs, is a [complete] solid orb on one side of which is the planet. That sphere moves with the proper motion of the planet, whose equator we conceive to be outside of the plane of the eccentric equator, intersecting the latter at the two mean distances. Then the diameter passing through the two mean distances is in the plane of the eccentric equator; [as for] the diameter passing through the [epicyclic] apex and perigee, one half [of it] will be in one direction and the other half [of it] in the other direction. If a line is drawn from the center of the inclined [orb] to the center of the epicycle and extended until it reaches the epicycle, then it necessarily intersects the diameter passing through the apex and the perigee at the center of the epicycle. Since this line is in the plane of the eccentric orb, the distance between this line and the diameter of the epicycle [passing] through the apex and perigee is equal to the inclination of the apex and the perigee, according to this illustration:

1. On this section, see F. Jamil Ragep, "Ibn al-Haytham and Eudoxus: The Revival of Homocentric Modeling in Islam," in *Studies in the History of the Exact Sciences in Honour of David Pingree,* edited by Charles Burnett, Jan P. Hogendijk, Kim Plofker, and Michio Yano (Leiden: E. J. Brill, 2004), 786–809.



[Figure 1]

[3] Then we conceive a second orb such that it encloses this [first] orb, and each of these two orbs shares the same center. This orb moves about two poles on the line that comes from the center of the inclined orb, with a motion like that of the epicycle center around the equant center. When this orb moves and carries the first orb with it, then necessarily [both] the apex and the perigee describe two circuits each of whose centers is on the line that comes from the center of the inclined [orb]. These are two small circles each of whose planes is perpendicular to the plane of the eccentric orb, like an [archery] target whose diameter is mounted on the circumference of a shield. So, two positions on [each] circle will be in the plane of the deferent orb, and, when the apex and perigee move along the circumference of [each] circle, they will be in the plane of the eccentric orb whenever they reach these two points. At the midpoint between these two points, they will be at the maximum inclination from the plane of the eccentric.

[4] There follows, however, from this motion a distortion, namely that since the entire epicycle moves with this motion, the diameter passing through the two mean distances goes out of the plane of the defer-

ent and makes a [complete] revolution during which the eastern half of the epicycle becomes western, and the western half becomes eastern. Then in order to rectify this distortion, we conceive another orb, which is the third orb enclosing these [previous] two orbs, in such a way that its center is the center of both orbs. Its two poles are at the two endpoints of the diameter of the epicycle orb that passes through the apex and perigee. Its motion is in the opposite direction of the second orb's motion but equal to it, so that by however much the equator of the epicyclic orb is displaced from its proper place by the motion of the second orb, this orb brings it back to its proper place, and the diameter of the mean distance always remains in the plane of the eccentric orb. However, the apex and the perigee continue to revolve on the above-mentioned circuits, since the poles of this orb are at the two endpoints of the diameter of the epicycle orb and the poles of the second orb are different from these two poles; the distance between each two of these four poles is equal to the radius of the circuit of the apex or of the perigee. Therefore, from these motions it follows that the apex half is always in one direction and the perigee half is in another direction opposite that of the first. In every revolution, the equator of the epicycle arrives twice at the plane of the eccentric equator and [then] passes through it in such a way that the directions interchange.

[5] As the epicycle center traverses the circumference of the inclined [orb] with a motion that is nonuniform with respect to the inclined center and uniform with respect to the equant center, then in the two quadrants that fall in the apogean half it is slower, while in the other two quadrants it is faster. Similarly, the apex and the perigee traverse [their] two circuits with a motion that is nonuniform with respect to the circuit's center but uniform with respect to a point other than the circuit's center that is within the circuit in a position having the status of the equant center, so that the movement of the apex on the circumference of this circuit is slower in two quadrants and faster in [the

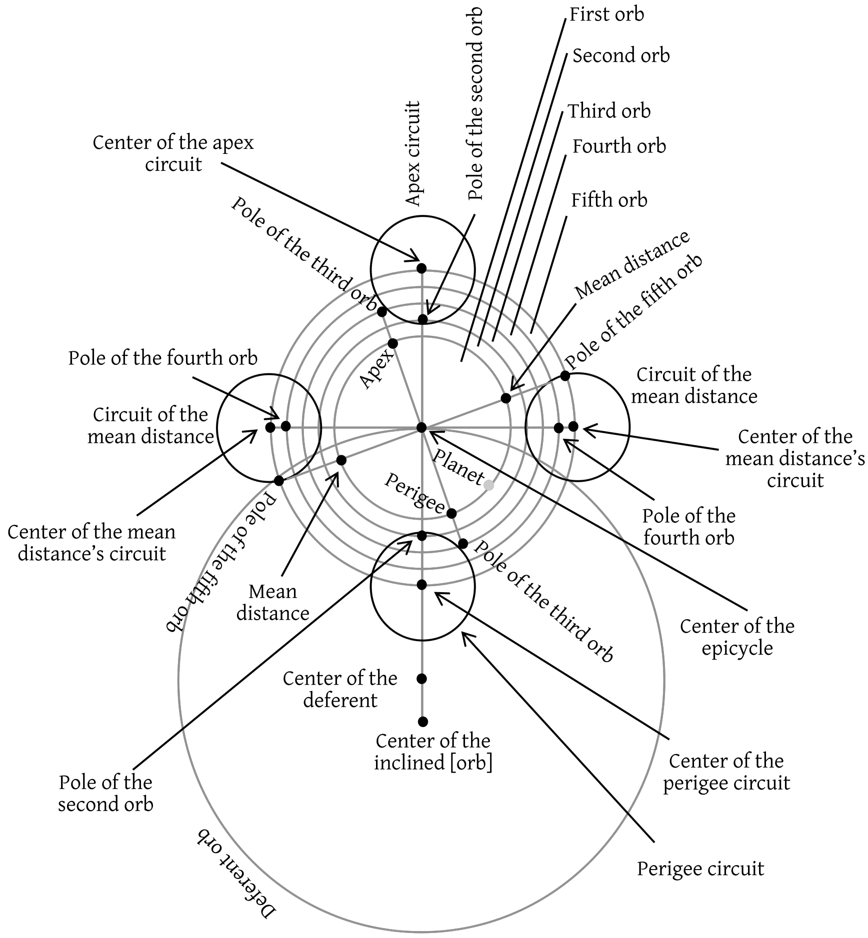
other] two quadrants. And likewise the movement of the perigee, so that the conformity with the motion of center is preserved.

[6] These two small circles are those that the author of *Muntahá al-idrāk* introduced when positing the bodies that are the principles of the motions, and he limited himself to that. Even though these two circles were first posited by Ptolemy in the *Almagest*, nevertheless since Ptolemy limited himself in all cases to circles, this case is consistent with other cases. [On the other hand,] someone who in other places posits bodies but here limits himself to circles is not observing the condition of consistency.

[7] For the two lower planets, to account for the inclination of the apex and the perigee, [Ibn al-Haytham] posited these same two orbs in addition to the epicycle orb; to account for the motion of the slant, he posited two other orbs. The first orb, which is the fourth orb for the epicycles of these [two planets], encloses the other three orbs. The two poles of this orb are two points on a line passing through the epicycle center in the plane of the deferent orb and intersecting at right angles the line that comes from the center of the inclined [orb]. When [this] orb moves, the diameter, which passes through the two mean distances, must necessarily move around these two poles. Thus, there results the motion of the slant, except that since the entire epicyclic equator moves, the apex and the perigee will become displaced from their proper places; the apex goes to the perigee's place and the perigee to the place of the apex. Therefore, a fifth orb encloses these four orbs; its two poles are the two endpoints of the line that passes through the two mean distances. Its motion is opposite and equal to the motion of the fourth orb, so that whatever is displaced from its proper place will return to its original position. Two circuits for the two mean distances result from the motion of the fourth orb, and these are the two small circles that intersect the plane of the deferent orb at right angles, like an [archery] target mounted on a shield,

such that each of the two circumferences are tangent at a point, and one plane intersects the other at right angles. The motion of these two [mean] distances on the circumference of these two circles varies, being faster in one half, slower in the other half, like the motion of center on the inclined orb. When the apex on its own circuit is at the maximum inclination from the inclined plane, the mean distance is in the inclined plane. And when the mean distance is at the maximum inclination from the inclined plane, the apex is in the inclined plane, such that these two latitudes follow in an alternate way. The illustration of the orbs of the epicycles of these two planets, to the extent they can be drawn in a plane, is as follows.

[8] The illustration of the orbs of the upper planets may also be known from this [illustration], if [we] only consider three orbs. This is the exposition of this treatise [of Ibn al-Haytham]—God is all-knowing.


First orb
Second orb
Third orb
Fourth orb
Fifth orb
Pole of the second orb
Apex circuit
Center of the apex circuit
Pole of the third orb
Mean distance
Pole of the fifth orb
Apex
Pole of the fourth orb
Circuit of the mean distance
Circuit of the mean distance
Center of the mean distance's circuit
Center of the mean distance's circuit
Planet
Perigee
Pole of the fourth orb
Pole of the fifth orb
Mean distance
Pole of the third orb
Center of the epicycle
Center of the deferent
Center of the inclined [orb]
Pole of the second orb
Center of the perigee circuit
Perigee circuit
Deferent orb

[Figure 2]