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Section [Three][1]
Concerning the Solution of the Doubt Arising with Regard to the Motion of the Center of the Lunar Epicycle on the Circumference of the Deferent, and the Uniformity of that Motion about the Center of the World

[1] In the Fifth Chapter of the Second Book, in the midst of [the description of] the configuration of the orbs of the Moon, this doubt occurs. As already pointed out, the same doubt [also] arises in the orbs of other planets, whereby it is assumed that the motion of the epicycle center is on the circumference of the deferent and its uniformity is with respect to the equant center. The upshot of the discussion is that since it is not possible for the motion of the celestial bodies to intensify or weaken, speed up or slow down, reverse direction or turn [from its course], except in relation to us, each sphere that moves must be such that its motion is uniform about its center. If that motion is considered with respect to a point other than its center, it will certainly be non-uniform, just as we have said for the Sun that its motion on the circumference of the eccentric is uniform about its center, but non-uniform about the center of the World. However, it is not reasonable to assume that a motion be non-uniform about its own center and uniform about a point other than its center, while maintaining those principles [regarding the motion of the orbs]. Therefore, anyone who investigates this science and who would posit for every motion an orb causing that motion must take the orbs to be such that

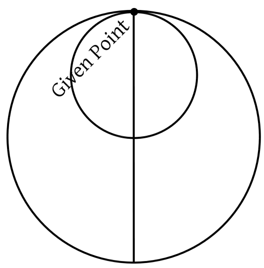
1. For a discussion of this chapter and a comparison with Naṣīr al-Dīn's handling of the Ṭūsī couple in his *Memoir on Astronomy*, see F. Jamil Ragep, "From Tūn to Toruń: The Twists and Turns of the Ṭūsī-Couple," in *Before Copernicus: The Cultures and Contexts of Scientific Learning in the Fifteenth Century,* edited by Rivka Feldhay and F. Jamil Ragep, 161–97 (Montreal: McGill-Queen's University Press, 2017).

the aims of this chapter—namely, the uniformity of motion about the center of the World along with the [continuous] equality of distance from the center of the deferent—be realized and that the motion of the orbs be in actual fact uniform. If [one] adds to or takes away from the number of the orbs, there is no objection against him; however, if he makes alterations in the models (*uṣūl*), which have been found through observation, or overlooks the conformity of some of the principles or premises, he misses the mark. Since Ptolemy, who set forth the principles with dispatch and who was the master of observation, did not take into account [physical] bodies and contented himself with setting forth lines and circles according to his purposes, he, and all those who follow his methods, have freed themselves from an obligation to this commitment. However, a group among the moderns, who have introduced an account of the corporeality of the orbs and a conception of the principles of the motions that they have found by observation, have undertaken this commitment and similar things.

[2] The solution of this doubt, *comme il faut*, is based upon geometrical lemmas. And because in the treatise nothing of that approach [i.e., geometrical reasoning] has been mentioned and [instead] a [summary] account of the problems was deemed sufficient without any geometrical proof, here too, conforming to that [approach], we will write down an indication, in so far as possible, of how to resolve the doubt in such a way that some of [our] intentions may be conceived—God willing. Now we say: in the cited chapter, it was shown that the motion of the epicycle center about the center of the World is uniform. It follows that the mover that gives it this motion has as its center the center of the World. Now the mover of the epicycle orb with this motion is the inclined orb of the Moon or another orb whose center is the center of the inclined orb. It is accepted that the distances of the epicycle center from the center of the World vary, but with respect to another point, such as the deferent center, they are equal. This can be [described]

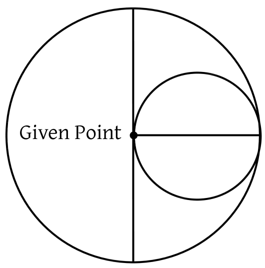
in this way: while the inclined orb gives the center of the epicycle a circular motion, another mover moves it rectilinearly toward the center of the World, such that [the epicycle] approaches the center of the World in one half of the inclined orb's revolution. Afterwards, it causes [the epicycle] to move, also rectilinearly, away from the center of the World in an upward direction, so that when the revolution of the inclined [orb] is completed, the epicycle center will return to its original position, which is the maximum distance from the center of the World. Thus, with this motion, in one half of the revolution, for example, it is closer to the center of the World and in the other half farther away. From the combination of the motion of the inclined orb with this motion, which we have assumed to be rectilinear, an eccentric circuit with respect to the center of the World will result for the epicycle center that is similar to a circle—even though, in fact, it is not a circle—and the uniformity about the center of the World will still be preserved.

[3] The rectilinear motion of the epicycle center away from the circumference of the inclined [orb] in the direction of its center, and afterwards its return on that same bearing until it reaches the circumference, without there resulting in any tearing or mending, or there being a rupture in the path of the circular motion, can be [described] in the way that we are going to mention. Before that, we will set forth a lemma, so that one may be better able to grasp the concept. We say: let us conceive of two circles such that the diameter of one is half that of the other, as in this illustration:



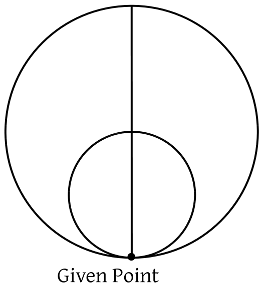
[Figure 1]

[4] At the point of tangency of both circles, let us draw a diameter that passes through both circles. And let us assume the larger circle to move in the counter-sequence [of the signs] and carry the smaller circle, and the smaller circle to move in the sequence [of the signs], the [smaller circle] carrying a given point, which in this illustration coincides with the point of tangency, in such a way that, when the larger circle has completed a rotation, the smaller circle will have completed two rotations. It follows from these two different motions that the given point moves rectilinearly on a diameter of the larger circle, never deviating from that line, such that it moves from this endpoint of the diameter to that [other] endpoint and from the latter endpoint to the former endpoint rectilinearly. For example, when the larger circle describes a quarter-rotation, the smaller circle describes half a rotation, [and] the given point [thereupon] coincides with the center of the larger circle and has traversed one-half the diameter of the larger circle, as in the following illustration:



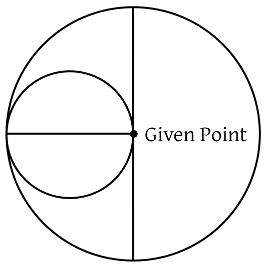
[Figure 2]

[5] Afterwards, when the larger circle has moved another quarter and the smaller circle a half, the diameter of the smaller circle will be coincident with the diameter of the larger circle, and the given point will coincide with the point of tangency, it having traversed the entire diameter, as in this illustration:



[Figure 3]

[6] Afterwards, the smaller circle comes to be in the other half of the large circle, and the given point returns rectilinearly, until the large circle describes a quarter [rotation] and the smaller circle a half, when once again, the given point coincides with the center of the large circle, having traversed half of the diameter, as in this illustration:

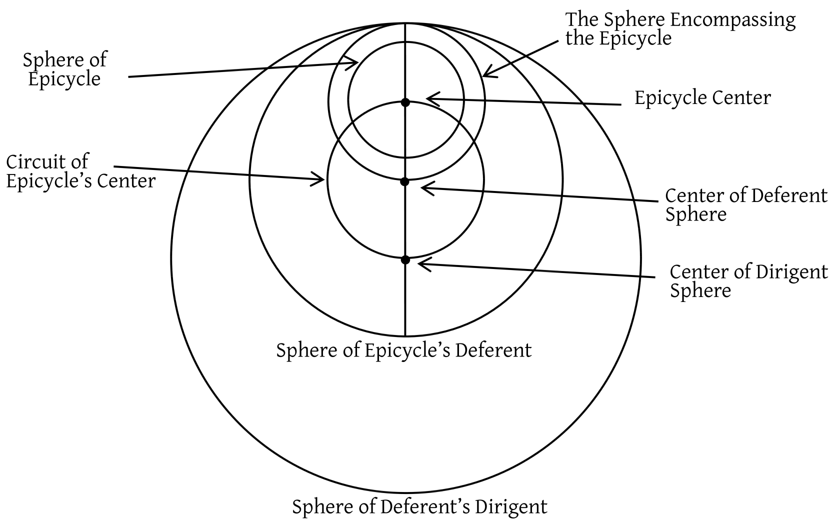


[Figure 4]

[7] Then when the large circle has described another quarter [rotation] and the small circle a half, the given point will reach the beginning endpoint of the diameter and comes to its own [initial] position. Thus, with one rotation of the large circle and with two rotations of the small circle, this point will have twice traversed the length of the diameter of the large circle rectilinearly: once from the first endpoint to the second endpoint and another time from the second endpoint to the first endpoint.

[8] If this lemma is understood, one may easily conceive for the lunar epicycle center a similar motion by means of bodies. This is such

that, in addition to the epicycle orb, we assume three spheres enclosing one another. The first is a sphere that encloses the epicycle and whose center is the epicycle center; we call it the enclosing sphere of the epicycle. Whatever thickness we assume for this orb will be suitable, since no defined limit is necessary for [the thickness]. The second is a sphere that encompasses the enclosing sphere and [it] has a center different from that of the latter, such that the circumferences of the two orbs touch at a single point; we call this the deferent sphere of the epicycle. The third is a sphere that encompasses the deferent sphere, just as the deferent encompasses the enclosing [orb], in such a way that the three spheres are tangent at that one point. And the radius of [this] dirigent sphere is equal to the sum of the eccentricity that we stated in the chapter on the Moon, i.e., 10;19, plus the radius of the epicycle, i.e., 5;15,[1] plus the amount of the thickness of the enclosing [orb] of the epicycle. The radius of the deferent is equal to half the eccentricity plus the radius of the epicycle plus the amount of the thickness of the enclosing [orb], as in this illustration:



[Figure 5]

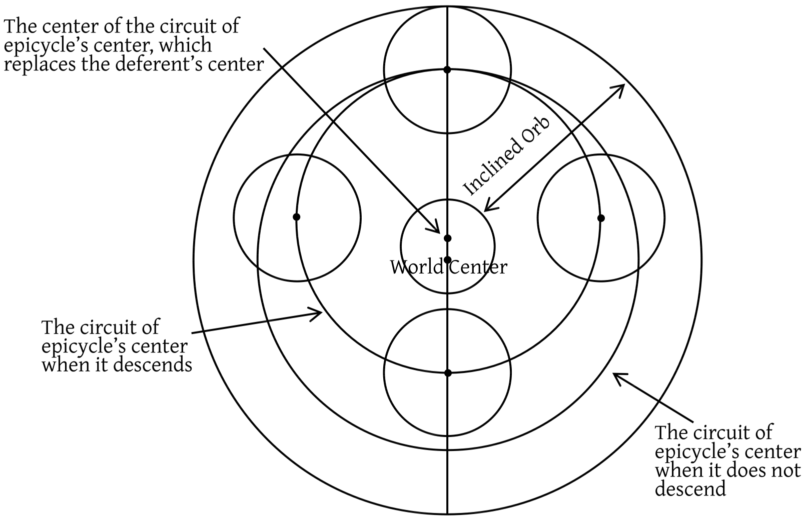
1. Edition should be corrected to read هـ يه.

[9] So the circuit of the epicycle center that results from the motion of the deferent passes through the center of the dirigent sphere, and its radius is equal to [half][1] the eccentricity, as has been outlined in black [in Figure 5]. Thus, as we have said, from the motion of the dirigent in one direction and the motion of the deferent in the opposite direction with twice that motion, it follows that the center of the epicycle will descend rectilinearly on the diameter of the dirigent in the amount of twice the diameter of its circuit. And since we have assumed the diameter of the circuit to be equal to the eccentricity, the rectilinear descent of the epicycle center will be in the amount of twice the eccentricity. Afterwards, also rectilinearly, it will ascend until reaching its original position, except that since the apex and [epicyclic] perigee are always aligned with the deferent center, it follows that the epicyclic apex[2] will deviate from the alignment with the dirigent diameter. After half a rotation of the dirigent, the epicycle diameter will be reversed, the apex at the bottom, the perigee at the top. Thus, we assume the epicycle's enclosing sphere to have a motion equal to the motion of the dirigent and in the same direction, so that the apex and the perigee will be brought back by the same amount it has deviated from its alignment with the dirigent diameter to its original position, thus always remaining coincident with the dirigent diameter. Thus, it will descend and ascend rectilinearly along the diameter. This being said, if we conceive this larger orb, namely the dirigent orb, to be embedded in the thickness of the inclined orb of the Moon, just as the epicycle is in the thickness of the deferent, and [if] the inclined [orb] moves uniformly about its own center, so that its rotation is completed with the rotation of the dirigent, there results from the circuit of the epicycle center a figure resembling a circle whose center is removed

1. Only MS K has half.

2. Ignoring the و in the edition.

from the center of the World by the amount of the eccentricity. As the inclined orb carries the epicycle with a circular [motion], while it gradually descends and approaches the center of the World until one half of the rotation of the inclined orb has been completed, the epicycle will have reached its maximum descent in the amount of twice the eccentricity, which is the thickness of the complementary [orb]. It will then be at the perigee, which is facing the original position, namely the apogee. It will ascend, again gradually, also on a circular figure, until reaching the original position. The epicycle center is always on the circumference of a nearly circular circuit, which is taken to replace the [Ptolemaic] eccentric of the Moon. Although the motion of the epicycle center is on the circumference of this [pseudo-] circle, it will [nevertheless] be uniform about the center of the inclined [orb]. One may conceive this concept from the following illustration:



[Figure 6]

The black line is the extent of the descent and ascent of the epicycle center; its maximum is in the amount of twice the eccentricity.[1]

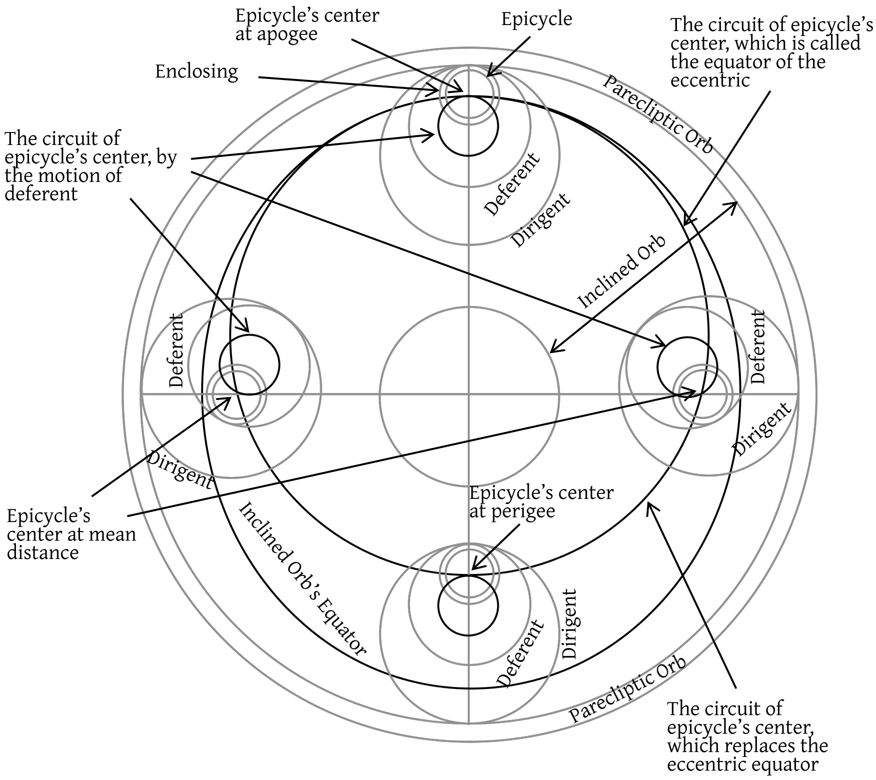
1. This additional sentence is in some of the manuscripts but not in our critical

[10] Now that this introduction has been laid down, six solid orbs for the Moon will be necessary: the first is the parecliptic whose motion is with the motion of the nodes in the counter-sequence [of the signs]; the second is the inclined orb whose motion is equal to the mean motion of Moon in the sequence [of the signs]; the third is the dirigent orb whose motion is equal to that of the Moon's motion of double elongation, which may be designated in whichever direction. Due to this motion, the lunar epicycle is at the apogee at conjunction and opposition, i.e., it touches the outer surface of the inclined orb, and is at the perigee at the two quadratures, i.e., it touches the concavity of the inclined orb; the fourth is the deferent orb of the epicycle whose motion is twice the motion of the dirigent and in the opposite direction; the fifth is the enclosing orb of the epicycle whose motion is in the same direction as the dirigent's motion and equal to it; the sixth is the motion of the epicycle orb with [the Moon's] proper motion, which in the upper half is in the counter-sequence [of the signs] and in the lower half in the sequence [of the signs], and the Moon is moved with this motion. The illustration of these orbs in relation to one another has been set down on another page.

[11] The doubt that has been brought up in [the case of] other planets may also be resolved in this way if the equant is taken to replace the inclined orb and the deferent to replace the eccentric. These orbs, motions, and bodies are not used by geometricians, who [instead] posit motions and explain the anomalies with lines and circles; rather, what Ptolemy has set forth regarding this matter is sufficient [for them]. However, anyone wishing to conceptualize how the motions are in accordance with observations while preserving philosophical principles needs to posit these orbs. This is the exposition of an answer to this difficulty to the best of [our] abilities on this occasion. However, with-

edition; for details, see the variants to this passage.

out presenting geometrical theorems, it is not possible to prove that the above motion is strictly rectilinear, that the path of the epicycle center is not a true circle but rather a quasi-circle, and that the deviation from its circularity does not produce a noticeable deviation in the positions of the Moon. Thus, in this place, we must limit [the discussion] to this extent.



[Figure 7]