How to cite this document:

Naṣīr al-Dīn al-Ṭūsī. *al-Risāla al-Muʿīniyya*, book 4, chapter 5. In F. Jamil Ragep, Fateme Savadi, Sajjad Nikfahm-Khubravan. *al-Risāla al-Muʿīniyya (al-Risāla al-Mughniya) and its Supplement*. Vol. II, *English Translation* (Tehran: Mirath Maktoob), 143–146.

CHAPTER FIVE
On Determining the Distances and Sizes of the Vacillating Planets

[1] For every wandering planet that is below the orb of Mars and that has some perceptible parallax, its parallax at the farthest distance is equal to the parallax of the wandering planet above it at the nearest distance. It therefore became known that the farthest distance of one is contiguous to the nearest distance of the one that is above it, and this same consideration was followed for the upper planets. Let us begin with Mercury.

[2] Mercury: The ratio of its diameter at the farthest distance to its diameter at the nearest distance has been found in theory to be as the ratio of 1 to $2+\frac{1}{3}+\frac{1}{4}$. Since the farthest distance of the Moon, which is the nearest distance of Mercury, has been found to be $64+\frac{1}{4}$, this amount was multiplied by $2+\frac{1}{3}+\frac{1}{4}$, the result being 166, which is Mercury's farthest distance. Its mean distance is the average of these two, i.e., 115—the radius of the Earth being 1. Therefore, in miles the nearest distance is 245,306, the mean distance 439,070 miles, and its farthest distance 633,788 miles.

[3] When Mercury's distances became known, Mercury's diameter at mean distance was found to be $\frac{1}{15}$ of the diameter of the Sun by precise observation and calculation, on the condition that the Sun be also at the mean distance. Since the farthest distance of the Moon is $64+\frac{1}{4}$, and the Sun's mean distance is 1,208, and the ratio of diameter to diameter is as the ratio of distance to distance, we decided to assume for the Earth an amount whose ratio to the distance of the Sun is as the ratio of the Earth's diameter to the Sun's diameter. The ratio of the Moon's diameter to the Earth's is 1 to $3+\frac{2}{5}$. We then multiplied the farthest distance of the Moon by $3+\frac{2}{5}$, the result being 218. Then, the ratio of

218 to 1,208 is as the ratio of the Earth's diameter to the Sun's, and this ratio is taken as the standard measure for all the planets.

[4] Since the ratio of Mercury to the Sun is the ratio of 1 to 15, and the ratio of Mercury's diameter to $\frac{1}{15}$ of the Sun['s] is as the ratio of the Sun's distance to Mercury's distance, therefore the ratio of $\frac{1}{15}$ of Mercury's distance to the Sun's distance is as the ratio of Mercury's diameter to the Sun's diameter. We divide Mercury's distance by 15, resulting in $7+\frac{2}{3}$. Therefore [the ratio of] Mercury to the Earth is as $7+\frac{2}{3}$ to 218. We divided 218 by it; one finds that Mercury to the Earth is as 1 to 28 plus a fraction. When both [values] are cubed, one finds that Mercury's size to the Earth's size is as 1 part to 22,000.

[5] Venus: The [proportional] difference between its diameter at the farthest distance and its diameter at the nearest distance in theory was found to be as 1 to7 minus a fraction. When this amount was multiplied by the farthest distance of Mercury—since Mercury's farthest distance is Venus's nearest distance—the result became 1,160. This is Venus's farthest distance, [which is] close to the Sun's nearest distance according to the requirement of the preceding calculation; thus, this is evidence of the correctness of the procedure. The mean distance [of Venus] according to this calculation is 663. Therefore, in miles [its] mean distance is 2,531,335, and [its] farthest distance is 4,428,880 miles.

[6] Venus's diameter at mean distance to the Sun's diameter at mean distance has been found in theory and by observation to be approximately $\frac{1}{10}$. The [measure in] parts of the mean distance, which is 663, was divided by 10, and $66+\frac{3}{10}$ was obtained; this is the [measure in] parts of Venus—the measure of the Earth being 218. When the [measure in] parts of the Earth was divided by this, the result was $3+\frac{1}{4}$; it [thus] was found that Venus's diameter to the Earth's diameter is as 1 to $3+\frac{1}{4}$. Both amounts were cubed; it was [now] found that [the ratio

of] Venus's size to the Earth's size is approximately as 1 to 34+$\frac{2}{3}$.

[7] Mars: The [proportional] difference between its size at the farthest and at nearest distances was found, in theory, as 1 to 7 minus something, like the case for Venus. The farthest distance of the Sun, which is the nearest distance of Mars, was multiplied by this amount; Mars's farthest distance was then 8,764 and the mean distance was 5,008. Thus, the mean distance is 19,120,544 miles and the farthest distance 332,460,952[1] miles. Mars's ratio to the Sun when both are at mean distance has been found to be 1 to 20. The mean distance was divided by 20, resulting in 250+$\frac{2}{5}$. This was divided by 218—the [standard measure in] parts of the Earth—resulting in 1 part and 7 minutes; thus, the Earth's diameter to Mars's diameter is as 1 [part] and 7 minutes. Both were cubed and it was found that the size of the Earth to the size of Mars is as 1 to 1+$\frac{1}{2}$.

[8] Jupiter: The [proportional] difference between its diameter at the farthest and at the nearest distance is as 1 part to 1 [part] and 31 minutes. Mars's farthest distance was multiplied by this amount, the farthest distance of Jupiter becoming 14,168. According to this standard, the mean distance is 11,466—the radius of the Earth being 1. The mean distance is 43,777,188 miles and its farthest distance is 542,093,424[2] miles. By sight, [the ratio of] its [apparent] size to the Sun['s], when both are at mean distance, is half of $\frac{1}{6}$. The mean distance was divided by 12, and the result was 955. [This] was divided by 218, resulting in 4+$\frac{1}{4}$+$\frac{1}{6}$. Thus, the Earth's diameter to Jupiter's diameter is 1 to this amount. Both amounts were cubed; it was [then] found that Jupiter's

1. The correct number should be 33,460,952, but because 332,460,952 is attested by our best witnesses, we have retained it in the text. We do not know the origin of the mistake, whether it is due to Ṭūsī or an early copyist.

2. The correct number should be 54,093,424; again, it is not clear whether the origin of the mistake is due to Ṭūsī or an early copyist.

size is $84+\frac{1}{4}+\frac{1}{8}$ times [the size of] the Earth.

[9] Saturn: The [proportional] difference between its diameter at the farthest and nearest distances is as 1 to $1+\frac{2}{5}$. When Jupiter's farthest distance is multiplied by this amount, it becomes 19,835, which is Saturn's farthest distance. According to this standard, the mean distance is 17,001. Thus, the mean distance is 44,909,818[1] miles and the farthest distance is 75,730,030 miles. [The ratio of Saturn's apparent size] to the Sun, when both are at mean distance, was found to be as $\frac{1}{2}$ of $\frac{1}{9}$. Saturn's mean distance was divided by 18, resulting in $944+\frac{1}{2}$. [This] was divided by 218—the [standard measure in] parts of the Earth—resulting in $4+\frac{1}{3}$. Therefore, the Earth's diameter to Saturn's diameter is 1 to $4+\frac{1}{3}$. When both amounts are cubed, it will be found that [the ratio of] the Earth's size to Saturn's size is as 1 to $81+\frac{1}{5}+\frac{1}{6}$.

CHAPTER SIX

On Determining the Distance and Sizes of the Fixed Stars

[1] Since all the fixed [stars] had been placed on one orb, their distance was taken to be the same distance, which is equal, according to the aforementioned standard, to Saturn's farthest distance. This [distance] has already been given in terms of the Earth's radius and in miles; in parasangs it is 25,243,343 parasangs. This is the limit of the distance of bodies for which humans have a way to know. When they considered the sizes of the [fixed stars], they classified them into six magnitudes, as has been stated, such that a star of the first magnitude is taken to be, for example, [the size of] a *dirham*, and that of the sixth magnitude to be [the size of] a *dāng* [i.e., one sixth of a *dirham*], according to this standard. This is only an extremely approximate estimation.

1. The correct number should be 64,909,818.