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Moon’s farthest distance from the Earth is 245,306 miles.

[3] If we want to know the amount of the height of the Earth’s shadow: since for a distance of $64 + \frac{1}{4}$ parts from the surface of the Earth, the radius of the shadow decreases by 5 digits, and [since] half of the whole base of the shadow is $20 + \frac{1}{2}$ digits, then this amount [of the shadow] comes to a point at 264 parts, this being the maximum distance of the shadow from the Earth. In miles it is 1,007,952 miles; and in parasangs, 335,984 parasangs. At this distance, the Earth’s shadow comes to a point. By this calculation, it is known that the shadow terminates at the nearest distance of Venus and expires in the thickness of [Venus’s] orb. This is the purport of this chapter—God is all-knowing of the Truth.

CHAPTER FOUR

On Determining the Size and Distances of the Sun

[1] When [the astronomers] observed carefully, [they found that] the Moon’s surface at its farthest distance is almost exactly equal to the Sun’s surface at its mean distance—and this estimation can be known by the observation of a solar eclipse. When there are two objects equal in sight but different in distance, the ratio of the diameter of one to another is as the ratio of the distance [of one] to the distance [of the other], as indicated by geometrical proof and the rules of the science of optics. The ratio of distance to distance is as the ratio of parallax to parallax counter-proportionally, i.e., the ratio of the Moon’s distance to the Sun’s distance is as the ratio of the Sun’s parallax to the Moon’s parallax.

[2] The [astronomers] observed the parallax of the two at this aforementioned distance as precisely as possible and found the Sun’s parallax at mean distance to be $1' 27''$ and the Moon’s parallax at its farthest distance to be $27' 10''$. Since the ratio of the Moon’s diameter to

the Sun's diameter is the same as the ratio of the Sun's parallax to the Moon's parallax, they divided the Sun's parallax by the Moon's parallax, the result being $18\frac{4}{5}$. Thus, it became known that the ratio of the Moon's diameter to the Sun's diameter is as the ratio of 1 to $18\frac{4}{5}$. It had already been known that the ratio of the Moon to the Earth is the ratio of 1 to $3\frac{2}{5}$. Therefore, the ratio of the Earth to the Sun is as the ratio of $3\frac{2}{5}$ to $18\frac{4}{5}$. If one divides this [latter] amount by the [former], the result will be $5\frac{1}{2}$. Therefore, the ratio of the Earth to the Sun is as the ratio of 1 to $5\frac{1}{2}$. The cube of 1 is 1; the cube of $5\frac{1}{2}$ is $166\frac{1}{4}+\frac{1}{8}$. Thus, it became known that the Sun is $166\frac{1}{4}+\frac{1}{8}$ times [as large as] the Earth.

[3] Furthermore, if the ratio of the Sun to the Moon is desired, cubing $18\frac{4}{5}$ will be approximately 6,645. So, the Sun is 6,645 times [as large as] the Moon. As for the Sun's distances, since the ratio of the Moon's diameter to the Sun's diameter is equal to the ratio of distance to distance, and the ratio of diameter to diameter is as the ratio of 1 to $18\frac{4}{5}$, then the mean distance of the Sun is $18\frac{4}{5}$ times the farthest distance of the Moon. Then, we multiplied $64\frac{1}{5}$, which is the farthest distance of the Moon, by $18\frac{4}{5}$ and obtained 1,208, which is the mean distance of the Sun, the radius of the Earth being 1.

[4] Ptolemy found the distance between the Sun's two centers to be $2\frac{1}{2}$ degrees—the radius of the precliptic at mean distance being 60 degrees—which he multiplied by $18\frac{4}{5}$ and obtained 47. If one adds this amount to 1,208, the result will be 1,255, which is the Sun's farthest distance. If one subtracts [47] from this amount [1,208], the remainder will be 1,161, which is the Sun's nearest distance. If one multiplies these amounts by the miles of the Earth's radius, one will obtain: 4,432,698 miles for the amount of the nearest distance; 4,612,144 miles for the mean distance; and 4,791,590 miles for the farthest distance. Therefore, [the distance] from the Earth to the Sun's mean distance is approximately 1,537,381 parasangs—God is all-knowing.