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## CHAPTER THREE

## On Determining the Distances of the Moon from the Earth

[1] [For] each amount that is to be determined, there needs to be a scale, such as the yardstick for surveying lands and determining the measurements of fabrics. For determining the [size of] bodies and distances, the practitioners of the discipline have made the Earth the scale; then by [the size of] its body they measure the [size of heavenly] bodies and by its radius the distances. The convention is to take the scale as one unit and measure other amounts in terms of its units.

[2] Since the ratio of the Moon's diameter to the Earth's is known, and the Moon's diameter at its farthest distance is approximately  $32'$  of the circumference of the inclined [orb], and the ratio of the circumference to the diameter is as the ratio of  $3+\frac{1}{7}$  to 1, then the ratio of the Earth's diameter to the diameter of the inclined [orb] is known, that being approximately 1 to 60. Therefore, the Moon's farthest distance on the eccentric orb from the Earth's surface is 59 degrees; and its farthest distance at apex and apogee, which is the limit of the Moon's distances, is  $64+\frac{1}{4}$  degrees. As the distance between the two centers is  $10^\circ 19'$ —the radius of the inclined [orb] or of the parecliptic being 60 degrees—the distance of the eccentric perigee from the equator of the inclined [orb], i.e., the thickness of the complementary [body], is double this amount, or  $20^\circ 38'$ , and the radius of the epicycle is  $5+\frac{1}{4}$  degrees. By subtracting these two amounts from 59, the remainder is  $33^\circ 7'$ , which is the Moon's nearest distance from the Earth—the radius of the Earth being 1 degree. Since the Earth's radius, according to Ptolemy's considerations, is 3,818 miles, which is approximately 1,273 parasangs, the Moon's nearest distance from the Earth's surface is 126,440 miles. Adding the miles of the Earth's radius to this amount, it becomes 130,258 miles, i.e., approximately 43,419 parasangs, and this amount is the radius of the world of generation and corruption. The

Moon’s farthest distance from the Earth is 245,306 miles.

[3] If we want to know the amount of the height of the Earth’s shadow: since for a distance of  $64 + \frac{1}{4}$  parts from the surface of the Earth, the radius of the shadow decreases by 5 digits, and [since] half of the whole base of the shadow is  $20 + \frac{1}{2}$  digits, then this amount [of the shadow] comes to a point at 264 parts, this being the maximum distance of the shadow from the Earth. In miles it is 1,007,952 miles; and in parasangs, 335,984 parasangs. At this distance, the Earth’s shadow comes to a point. By this calculation, it is known that the shadow terminates at the nearest distance of Venus and expires in the thickness of [Venus’s] orb. This is the purport of this chapter—God is all-knowing of the Truth.

#### CHAPTER FOUR

##### On Determining the Size and Distances of the Sun

[1] When [the astronomers] observed carefully, [they found that] the Moon’s surface at its farthest distance is almost exactly equal to the Sun’s surface at its mean distance—and this estimation can be known by the observation of a solar eclipse. When there are two objects equal in sight but different in distance, the ratio of the diameter of one to another is as the ratio of the distance [of one] to the distance [of the other], as indicated by geometrical proof and the rules of the science of optics. The ratio of distance to distance is as the ratio of parallax to parallax counter-proportionally, i.e., the ratio of the Moon’s distance to the Sun’s distance is as the ratio of the Sun’s parallax to the Moon’s parallax.

[2] The [astronomers] observed the parallax of the two at this aforementioned distance as precisely as possible and found the Sun’s parallax at mean distance to be  $1' 27''$  and the Moon’s parallax at its farthest distance to be  $27' 10''$ . Since the ratio of the Moon’s diameter to