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CHAPTER TWO
On Determining the Ratio of the Moon's Size to the Earth

[1] When one reflects on lunar eclipses that are latitudinally and directionally equal, but at varying distances from the Earth, [it can be seen that] the higher [i.e., farther away from the Earth] the Moon is during [the eclipse], the less the duration [of the eclipse], and the closer it is to the Earth the longer the duration. [Now,] this closeness and farness in distance can be due only to the epicycle orb, because a lunar eclipse always occurs at the farthest distance of the eccentric orb. This indicates that the farther the Earth's shadow is from the Earth the narrower it is. Thus, it resembles a pinecone in shape, whose base is the Earth; for if [the shadow] were wider the farther away it is, the duration of the eclipse at the [shadow's] apex would be longer, but it is not; and if it were of a uniform width [and of a] cylindrical shape, the duration [of eclipses] at all distances would be the same, but it is not. Since the [Earth's] shadow is narrower the farther it is from the Earth, the Sun is larger than the Earth, for if it were smaller than the Earth, the shadow would be wider the farther it is from the Earth. If the Sun were equal to the Earth, the shadow would be cylindrical. Since the shadow is a cone with the Earth as its base, no circle can be assumed on that cone larger than the equator of the Earth, which is [its] base. Since at the place of the Moon the shadow is smaller than the equator of the Earth and completely obscures the body of the Moon, the Moon is smaller than the Earth. Therefore, based on this consideration, it has become known that the Sun is larger than the Earth, and the Moon is smaller than the Earth. Since the Sun is larger than the Earth, the shadow gets smaller until it reaches a point where it vanishes.

[2] For determining the size of the Moon and the shadow, two lunar eclipses were sought during both of which the Moon would be at

the apex of the epicycle; during one, a quarter of the diameter of the Moon's surface was eclipsed, while during the other, half [of the diameter was eclipsed]. In the first eclipse, the latitude of the Moon was found to be 49 minutes plus a fraction, and in the second eclipse, 41 minutes plus a fraction. Therefore, it was determined that for every 8 minutes minus a fraction that the latitude decreases, one quarter of the size of the Moon's diameter is added in eclipse. One quarter of the Moon's diameter was taken to be 3 digits, since the whole diameter is taken to be 12 digits. Since in the second eclipse half of the diameter was eclipsed, the shadow circle had passed through the center of the Moon; thus, the amount of the latitude of the Moon was [equal to] the radius of the shadow circle, inasmuch as the center of the shadow circle always adheres to the zodiacal equator, facing the center of the body of the Sun. When the Moon's latitude in the second eclipse, which is equal to the radius of the shadow, is multiplied by 3 digits and divided by 8 minutes minus a fraction, the result will be fifteen and a half digits, which is the radius of the shadow at the Moon's epicycle apex—the diameter of the Moon being 12 digits.

[3] Thereafter, two other eclipses at perigee were sought, such that—as previously stated—one quarter of the Moon's diameter would be eclipsed in one, and half in the other. Using the same method mentioned, the radius of the shadow circle at the perigee was determined and found to be 16+$\frac{2}{6}$ digits. It was then determined that when the shadow was closer to the Earth by the amount of a diameter of the epicycle orb, the diameter of the shadow increased by $\frac{5}{6}$ of a digit, because between the first two eclipses and the second two eclipses was a difference of not more than the size of the diameter of the epicycle—with no conceivable difference due to the eccentric.

[4] Since the radius of the epicycle of the Moon, as has been stated, is 5+$\frac{1}{4}$ degrees—the radius of the inclined [orb] being 60 degrees—and the farthest distance of the eccentric is tangent to the surface of the

inclined [orb], considering circles, not bodies, then [the distance] from the apex of the epicycle orb to the center of the Earth will be $65+\frac{1}{4}$ degrees by this standard [of measurement]. This is the axis of the shadow cone. Since the radius of the epicycle is $5+\frac{1}{4}$ degrees, the diameter will be $10+\frac{1}{2}$ degrees. It has been determined that for every $10+\frac{1}{2}$ degrees that the shadow is closer [to the Earth], its radius increases by $\frac{5}{6}$ of a digit. [Thus] at this distance that the apex has from the Earth, the radius of the shadow increases by [the amount of] 5 digits plus a fraction. When this amount is added to the $15+\frac{1}{2}$ digits that were found when [the Moon was] at the apex of the radius of the epicycle, the result is the radius of the base of the shadow [cone], and this is equal to the radius of the Earth. Therefore, the diameter of the Earth is approximately 41 digits—the diameter of the Moon being 12 digits.

[5] When 41 is divided by 12, the result is $3+(2+\frac{1}{2})\times\frac{1}{6}$. Therefore, the ratio of the diameter of the Moon to the diameter of the Earth is 1 to $3+(2+\frac{1}{2})\times\frac{1}{6}$, [whereas] in Ptolemy's calculation it has been given as $3+\frac{2}{5}$. In Book Twelve of his work, Euclid proved that the ratio of the cube of the diameter of one sphere to the cube of the diameter of another sphere is equal to the ratio of the size of [the first] sphere to the size of that other sphere. If we cube the diameter of the Moon, 1 [multiplied] by 1 by 1, it will still be 1; and if we cube the diameter of the Earth, $3+\frac{2}{5}$ [multiplied] by $3+\frac{2}{5}$ by $3+\frac{2}{5}$, it will be $39+\frac{1}{4}$. This will the ratio of the [size of the] Moon to the Earth, that is, [the size] of the Moon to [the size of] the Earth is as 1 to $39+\frac{1}{4}$, which is what is required. If anyone wants to determine the surface area of the Moon, its diameter and [the size of] its body in parasangs, miles and cubits, it is possible, since these amounts are known for the Earth—God is all-knowing.