How to cite this document:

Naṣīr al-Dīn al-Ṭūsī. *al-Risāla al-Muʿīniyya*, book 3, chapter 12. In F. Jamil Ragep, Fateme Savadi, Sajjad Nikfahm-Khubravan. *al-Risāla al-Muʿīniyya (al-Risāla al-Mughniya) and its Supplement*. Vol. II, *English Translation* (Tehran: Mirath Maktoob), 132–134.

start at sunrise and reach their [upper] limit when the Sun is at the zenith. If the gnomons are perpendicular to the horizon, their shadows are called secondary shadows, which reach their [upper] limit at sunrise, and when the Sun is at the zenith it is diminished. Thus, the [lower] limit of one shadow is like the [upper] limit of the other shadow. For this reason, the primary shadow of any altitude is equal to the secondary shadow of the complement of that altitude. Because most shadows are secondary, and those are more visible, such shadows are called regular shadows, while the primary shadow is [called] a reversed shadow. The primary shadow is used in astronomical operations, and its measuring scale is taken to be 60 degrees, while some take it to be 1 degree. The secondary shadow is used in determining time, and its measuring scale is divided sometimes into seven divisions, or into six and a half divisions, that are called "feet"; sometimes into twelve divisions called "digits"; or sometimes into sixty [divisions] called "parts."

[4] In the fourth clime, the shortest noon shadow is the one cast at the beginning of Cancer, and the longest shadow is at the beginning of Capricorn. In other climes, it is according to altitudes, a bit of which has been explained in the preceding chapters—God is all-knowing.

CHAPTER TWELVE
On Determining the Meridian Line and Azimuth of Localities

[1] The meridian line is a line assumed on the surface of the Earth parallel to the [celestial] meridian circle. The line perpendicular to that line, which necessarily is parallel to the prime vertical [lit., circle of the initial azimuth], is called the east-west line. There are many ways to determine the meridian line, but the most famous of all is the Indian Circle. It is [constructed] such that on level land a pole is inserted.

Then its perpendicularity is checked by drawing a circle at the center of which the pole stands erect. Then the distances of three positions on the circumference of the circle from the vertex of the pole are measured. If they are equal, the pole is erect at right angles; otherwise it is inclining to one side. Then, at the beginning of the day, when the shadow has begun to decrease and intersects the circle, one watches to see at which point it enters the circle. At the end of the day, [one watches to see] at which point it exits. One draws a straight line between those two points, then extends a straight line from the center of the circle to the midpoint of that line. That line is the meridian [line], and the line perpendicular to it is the east-west line.

[2] If desired, one can mark the shadow of that gnomon at two times of equal altitudes on both sides of midday. From the base of the gnomon, one separates equal amounts from both shadows and draws a line connecting the two separation [points] so as to form an isosceles triangle from the two sides of the shadows and the produced line. Then from the midpoint of this line, a line is drawn to the base of the gnomon, which is the meridian line. Then that line, as we have said, corresponds to the meridian circle, and the east-west line corresponds to the prime vertical. When the point of intersection of these two lines is made a center [about which] a circle of any desired distance is drawn, that circle corresponds to the horizon circle.

[3] As for the azimuth of localities, it is an arc of the horizon circle between the north or south point and the intersection of the horizon circle with one circle of the altitude circles passing through the zenith point of another town. When two towns are longitudinally equal but latitudinally different, those two towns have no azimuth with respect to one another, the meridian line being their azimuth. This means, in the town whose latitude is less, one simply should face the north point [to face the place of greater latitude], while in the other town [of greater latitude one should face] the south point. As for two towns

whose latitude is the same but whose longitudes are different, it is generally thought that one must face east or west, but it is not so because towns of equal latitude are [on a circle] parallel to one of the day-circles, not parallel to [one of the] great circles, whereas the east-west line is parallel to one of the great circles. Therefore, the azimuth of such localities is inclined toward the north from the east or west. The calculation of the azimuth of localities belongs to practical books.

[4] What is needed most is the determination of the azimuth of Mecca. The longitude of Mecca has been given as 77°10′, and its latitude as 21°40′. Then, when the Sun is at either one of these two degrees, 7°20′ Gemini or 22°40′ Cancer, it will transit the zenith of the people of Mecca. If the longitude of any given town is a greater longitude than that of Mecca and the equinoctial moves by the amount of the longitudinal difference from noon, or if the [town's] longitude is less, and the same amount [of equinoctial movement equivalent to the longitudinal difference] will remain until noon, it will [then] be noontime for the people of Mecca, the direction of the shadow will be the direction line, and the direction of the Sun will be the azimuth of Mecca. If the altitude of the Sun at that time is determined by observation, the azimuth of Mecca will be known. As this is sufficient, let us end this Part here—God is the Provider of succor.