How to cite this document:

Naṣīr al-Dīn al-Ṭūsī. *al-Risāla al-Muʿīniyya*, book 3, chapter 11. In F. Jamil Ragep, Fateme Savadi, Sajjad Nikfahm-Khubravan. *al-Risāla al-Muʿīniyya (al-Risāla al-Mughniya) and its Supplement*. Vol. II, *English Translation* (Tehran: Mirath Maktoob), 131–132.

verting them to one another belong to books of praxis. This much on the knowledge of the true sense of the years and months is sufficient.

CHAPTER ELEVEN
On Understanding Shadows and Their Circumstances with Respect to Altitudes

[1] From the preceding chapters it has been ascertained that the [solar] altitude at noon, which is the maximum altitude of the Sun, is the amount of the Sun's declination plus local colatitude if the Sun is in the direction of the visible pole of the equinoctial, or the amount of the excess of local colatitude over the declination if it is in the other direction. [It has also been ascertained that] there is a shadow corresponding to every [solar] altitude. Just as the upper limit of the [Sun's] altitude is 90 degrees, and its lower limit is when the luminary is on the horizon, likewise, the upper limit of a shadow is infinity, and its lower limit is when there is no shadow at all [i.e., at noon]. Other shadows [in between] are in proportion to the altitude [of the Sun].

[2] The shadow of a gnomon is a line drawn from its base to the end of a line from the body of the luminary through the gnomon's tip to the surface on which that gnomon is standing erect. The hypotenuse of the shadow is a line from the gnomon's tip to that surface, as a part of the aforementioned line. Therefore, the height of any gnomon, the shadow, and the hypotenuse of the shadow all three lines [together] form a right triangle, the subtense of the right angle being the hypotenuse of the shadow.

[3] Gnomons are either perpendicular to the plane of the horizon or [perpendicular] to a plane that is perpendicular to the horizon, i.e., they are parallel to the horizontal plane. Then, if the gnomons are parallel to the horizon, their shadows are called primary shadows; they

start at sunrise and reach their [upper] limit when the Sun is at the zenith. If the gnomons are perpendicular to the horizon, their shadows are called secondary shadows, which reach their [upper] limit at sunrise, and when the Sun is at the zenith it is diminished. Thus, the [lower] limit of one shadow is like the [upper] limit of the other shadow. For this reason, the primary shadow of any altitude is equal to the secondary shadow of the complement of that altitude. Because most shadows are secondary, and those are more visible, such shadows are called regular shadows, while the primary shadow is [called] a reversed shadow. The primary shadow is used in astronomical operations, and its measuring scale is taken to be 60 degrees, while some take it to be 1 degree. The secondary shadow is used in determining time, and its measuring scale is divided sometimes into seven divisions, or into six and a half divisions, that are called "feet"; sometimes into twelve divisions called "digits"; or sometimes into sixty [divisions] called "parts."

[4] In the fourth clime, the shortest noon shadow is the one cast at the beginning of Cancer, and the longest shadow is at the beginning of Capricorn. In other climes, it is according to altitudes, a bit of which has been explained in the preceding chapters—God is all-knowing.

CHAPTER TWELVE
On Determining the Meridian Line and Azimuth of Localities

[1] The meridian line is a line assumed on the surface of the Earth parallel to the [celestial] meridian circle. The line perpendicular to that line, which necessarily is parallel to the prime vertical [lit., circle of the initial azimuth], is called the east-west line. There are many ways to determine the meridian line, but the most famous of all is the Indian Circle. It is [constructed] such that on level land a pole is inserted.