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ning of Capricorn.

[8] As for the hours of daytime and nighttime, they are of two sorts: one being equal, the other unequal. Seasonal [temporal] hours are also unequal. Equal hours are those that divide a nychthemeron into twenty-four equal divisions, each division being an hour. Therefore, when daytime of a [nychthemeron] is longer, the number of daytime hours increases; and when daytime is shorter, the number of hours decreases. The measure of hours is always equal, which is 15 degrees plus a bit of a revolution of the equinoctial.

[9] Seasonal [temporal] hours are those that divide the daytime, whether it be long or whether it be short, into twelve divisions, and so too the nighttime, each division being called an hour. Therefore, the measure of daytime hours differs from the measure of nighttime hours; the measure of one daytime hour along with the measure of one nighttime hour together equal the measure of two equal hours. In habitations at the equator there is no difference between equal and unequal hours—and God is all-knowing of the Truth.

## CHAPTER TEN

### On Determining the Year, Month, Calendar, Intercalation, and Their Like

[1] The basis of the month is from the visibility of the crescent to the full Moon until once again becoming imperceptible at the new Moon. Since this [cycle] is completed in approximately thirty days, and a year is completed in approximately twelve of these cycles, therefore the yearly cycle has been set to be twelve months and the monthly cycle to be thirty days. This situation also conforms to the twelve zodiacal signs and [their] having thirty degrees each. Since the best known of the planets and celestial bodies are the two luminaries, most nations

have considered the cycles of one of these two luminaries for setting up their months and years, with some having taken both into consideration. A year then is either solar or lunar, each of which may be either true or conventional.

[2] A solar year is [a period of time during] which the Sun leaves a [particular] point on the zodiacal orb and makes a full revolution, reaching that [same] point again. This occurs over a period of approximately  $365+\frac{1}{4}$  days.

[3] A true solar [year] is based on the [actual] revolutions of the Sun, not the number of days and months; one such is the Malikī year, wherein New Year's Day [Nawrūz] is set when the Sun reaches Aries. The months of this calendar are conventional, though if months were also based on the beginning of the zodiacal signs, the months would be true [solar months]. Since, however, the convention is to set months to be thirty days each, five days are left over, and they are called the stolen five [i.e., the epagomenal days]. Every few years, the fractions in excess [of 365] add up to a day, which is called the intercalary day.

[4] A conventional solar year is based on a number [of days] close to the true amount, like [that of] the Byzantines [Rūmiyān], who take the year to be exactly  $365+\frac{1}{4}$  days and thus every four years one intercalary day is added. They [i.e., the Byzantines] have distributed the stolen five days at the beginnings of the months, and therefore their year is never more than 366 days or less than 365 days. Some of their months have thirty [days] and some thirty-one. Since seven months have thirty-one [days], Shubāṭ has been set to be twenty-eight [days] or, during a leap year, twenty-nine. There is no rationale for such a set up. The Persians have set their year to be exactly 365 days, so that they do not have to take intercalation into consideration. [Their] months are each thirty days, with the stolen five [days added] at the end of the year. In olden times, they had an intercalary month every 120 years. Therefore, their

years, with that intercalation, became equal to the Byzantine years.

[5] A lunar [year] is [defined as] the Moon's reaching the Sun twelve times, which is achieved over approximately  $354 + \frac{1}{5} + \frac{1}{6}$  days. Each cycle of these twelve cycles is a month. This state of affairs will be a true [lunar month] if one of the Moon's positions vis-à-vis the Sun is made the starting point; when it reaches that position [again], it is counted as one month, as with the Arabs, who have established the visibility of the crescent as the beginning of the months. Their calendar is true lunar, with regard to both years and months.

[6] A conventional [lunar calendar] would take into consideration days and months, not consideration of the Moon's motion. This is the custom of those practitioners of arithmetic and astronomers, who take the year to be  $354 + \frac{1}{5} + \frac{1}{6}$  days; and from the beginning of Muḥarram, they take one month to be thirty days, and one month to be twenty-nine days until the end of the year. To account for the  $\frac{1}{5} + \frac{1}{6}$ , in every thirty years they intercalate eleven times and consider Dhū al-Ḥijja to be a full thirty days. This calendar is conventional with regard to both months and years. The years and months of the Jews are composed of lunar and solar. Their months are conventional, and every three years or two years an intercalary month is added so that the cycles of their years conform to the cycles of solar years. This convention is close to that of conventional solar years.

[7] It is for every nation to establish its own convention according to [its] judgment and want. Every nation sets the starting point of its calendars to be the beginning of a people or dynasty, or the occurrence of a great and famous event to which they relate their years and their months, such as the Arabs [who have chosen] the emigration of the Prophet—peace be upon him—the Byzantines the kingship of Alexander, son of Philip, and the Persians [the kingship of] Yazdgird, son of Shahryar. Precise knowledge of the principles of calendars and con-

verting them to one another belong to books of praxis. This much on the knowledge of the true sense of the years and months is sufficient.

## CHAPTER ELEVEN

### On Understanding Shadows and Their Circumstances with Respect to Altitudes

[1] From the preceding chapters it has been ascertained that the [solar] altitude at noon, which is the maximum altitude of the Sun, is the amount of the Sun's declination plus local colatitude if the Sun is in the direction of the visible pole of the equinoctial, or the amount of the excess of local colatitude over the declination if it is in the other direction. [It has also been ascertained that] there is a shadow corresponding to every [solar] altitude. Just as the upper limit of the [Sun's] altitude is 90 degrees, and its lower limit is when the luminary is on the horizon, likewise, the upper limit of a shadow is infinity, and its lower limit is when there is no shadow at all [i.e., at noon]. Other shadows [in between] are in proportion to the altitude [of the Sun].

[2] The shadow of a gnomon is a line drawn from its base to the end of a line from the body of the luminary through the gnomon's tip to the surface on which that gnomon is standing erect. The hypotenuse of the shadow is a line from the gnomon's tip to that surface, as a part of the aforementioned line. Therefore, the height of any gnomon, the shadow, and the hypotenuse of the shadow all three lines [together] form a right triangle, the subtense of the right angle being the hypotenuse of the shadow.

[3] Gnomons are either perpendicular to the plane of the horizon or [perpendicular] to a plane that is perpendicular to the horizon, i.e., they are parallel to the horizontal plane. Then, if the gnomons are parallel to the horizon, their shadows are called primary shadows; they