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northerly cross before the degree and the southerly after the degree; for those between the first of Cancer and Capricorn, it is the opposite.

[4] As for degrees of rising and setting, wherever the pole of the zodiacal orb is on the horizon, at that time the degree of all that rise or set is its degree of rising or setting. When one pole of the zodiacal orb is above the Earth, the rising of every star in the direction of that pole is before the degree, and [its] setting is after the degree. This is because from that direction when a latitude circle is imagined passing through the two points of the ascendant and descendant, the half that is above the Earth will be in the direction of the visible pole; thus, every part in that direction either will have risen before the degree or will not yet have set. The half below the Earth will be in the direction of the invisible pole, and parts in that direction either will have set before the degree or will have not yet risen. The rising and setting of parts at the equator are like their transits of the meridian, because the horizon at the equator is one of the meridian circles—God is all-knowing.

CHAPTER NINE
On Determining Day and Night, Dawn and Dusk, Unequal and Equal Hours, etc.

[1] Since day and night arise from the motion of the equinoctial, and the Sun moves in the opposite direction, therefore the period of one nychthemeron, which is from the Sun reaching a given point until it reaches that point a second time by the diurnal motion, is one revolution of the equinoctial plus the Sun's motion. Since the Sun's motion is not uniform, and the rise of parts on the zodiacal orb with parts of the equinoctial are not in conformity, differences occur in the periods of nychthemerons from two aspects: one due to the difference in the diurnal courses of the Sun, and the other due to the difference between equal degrees and degrees of co-ascension. Therefore, a mean

day is the amount of a revolution of the equinoctial with the addition of the Sun's daily mean, while a true day is the amount of one revolution plus the Sun's [actual] course during that revolution in terms of the equinoctial co-ascension. The difference between the mean and the true, which is compounded of the two aforementioned differences, is called the equation of the nychthemeron. Although this difference is not perceptible over one day or two days, it can be perceived over many days.

[2] The maximum difference between the Sun's mean and its true position is in the amount of the equation. Since the equation is additive in one half and subtractive in the [other] half, the maximum difference between mean and true days is twice the equation. The maximum difference between equal degrees and the degrees of co-ascension is $2+\frac{1}{2}$ degrees; since this is sometimes additive and sometimes subtractive, the maximum difference between true and mean days is therefore 5 degrees. Only rarely, however, are these two differences completely compounded together, for when one reaches maximum the other is usually less than maximum.

[3] As for the difference that arises from the equation of the Sun, it is subtractive in one half of the orb where the apogee is at the midpoint of that half, and additive in the other half. The equal degrees are greater than the degrees of co-ascension in the two quarters whose midpoints are the vernal and autumnal equinoxes, and smaller in the other two quarters. Thus, since at this time the Sun's apogee is at the end of Gemini, both increases come together in the quarter whose midpoint is the winter solstice.

[4] Since one specific day must be assumed when the mean and true coincide, so that every difference can be determined with respect to that day, and [since] for any given part other than the two ends of this quarter the equation is sometimes additive and sometimes subtrac-

tive, the practitioners of the discipline of the stars have assumed that specific part to be in Aquarius, so that the equation of the nychthemeron is always subtractive from mean days and additive to true days. Had they assumed it to be a part in Scorpio, it would have been the other way around. When one revolution of the Sun is completed, mean and true days will be at their original state, and the difference will be eliminated. Such is the equation of the nychthemeron.

[5] The beginning of a nychthemeron would naturally be placed at the beginning of daytime, except if mathematicians take it to be from the beginning of daytime or the beginning of nighttime, then another difference would be added to the equation of the nychthemeron. That difference [comes from] co-ascensions, which differ by localities, and add to or subtract from the beginning of day and night the amount of half the increase or decrease in the length of the day, [which is] due to the difference in the divisions of the day-circles. On the other hand, since the beginning of a nychthemeron is made to be the passage of the Sun across a circle by which all day-circles are divided with an equal proportion, such as the meridian circle, this difference disappears. Thus, for this reason mathematicians take the beginning of a nychthemeron to be noon and calculate the true position according to that time. Those who are not occupied with such calculations make the beginning of a nychthemeron the beginning of daytime, as do the Persians. The Arabs, on the other hand, because the beginning of their months is according to crescent visibility, make the beginning of a nychthemeron to be the beginning of nighttime.

[6] The beginning of daytime is when the Sun reaches the horizon circle and not the rise of dawn. The beginning of nighttime is when the Sun reaches the horizon and not the setting of dusk. As for dawn, which is the light of the Sun when it approaches the horizon, it has different circumstances due to the shape of the Earth's shadow, as it is in the form of a circular cone, as has been explained. So, when the Sun

is near the nadir, the apex of the cone is near the zenith; thus because of the accretion of darkness, the light of the Sun, which is at the edges of the Earth and encompasses the shadow cone, is not perceptible. Afterwards, as the Sun approaches the horizon and the cone inclines toward the west, an elongated light appears from a narrow slit on the side toward the east. That light is above the horizon, because the lines extending from the observer's position, i.e., the surface of the Earth, to the horizon are longer than those extending to the surface of the cone from above the horizon, as has been established by geometrical proofs. Therefore, first dawn is elongated, and its base, which is connected to the horizon, is dark; hence it is called "false." Afterwards, as the cone inclines more, the horizon becomes illuminated and the light spreads, which is "true dawn." After that, the horizon turns red, on account of the intensity of light, until the Sun rises. The situation of dusk is similar, only in reverse: first redness, then extensive whiteness, then elongated whiteness.

[7] By testing and observation, it has been determined that the beginning of dawn and the end of dusk occur when the altitude of [the degree directly opposite] the Sun or the Sun's depression below the horizon reaches the amount 18 degrees. Therefore, in habitations whose colatitudes are less than the obliquity plus 18 degrees, their dawn will be continuous with their dusk, and dusk continuous with dawn, when the Sun reaches degrees where the sum of the declination and the local colatitude surpasses 72 degrees. Since the Sun's depression being 18 degrees below the Earth is like its altitude in its [directly opposite] degree above the Earth, then in the oblique horizons the period of dawn and dusk is greater in the one half of the zodiacal orb that is in the direction of the local latitude than is the period of dawn and dusk in the other half. For example, in the fourth clime the longest period of dawn, which amounts to two hours, is at the beginning of Cancer, and the shortest period, which is one hour plus a fraction, is at the begin-

ning of Capricorn.

[8] As for the hours of daytime and nighttime, they are of two sorts: one being equal, the other unequal. Seasonal [temporal] hours are also unequal. Equal hours are those that divide a nychthemeron into twenty-four equal divisions, each division being an hour. Therefore, when daytime of a [nychthemeron] is longer, the number of daytime hours increases; and when daytime is shorter, the number of hours decreases. The measure of hours is always equal, which is 15 degrees plus a bit of a revolution of the equinoctial.

[9] Seasonal [temporal] hours are those that divide the daytime, whether it be long or whether it be short, into twelve divisions, and so too the nighttime, each division being called an hour. Therefore, the measure of daytime hours differs from the measure of nighttime hours; the measure of one daytime hour along with the measure of one nighttime hour together equal the measure of two equal hours. In habitations at the equator there is no difference between equal and unequal hours—and God is all-knowing of the Truth.

CHAPTER TEN
On Determining the Year, Month, Calendar, Intercalation, and Their Like

[1] The basis of the month is from the visibility of the crescent to the full Moon until once again becoming imperceptible at the new Moon. Since this [cycle] is completed in approximately thirty days, and a year is completed in approximately twelve of these cycles, therefore the yearly cycle has been set to be twelve months and the monthly cycle to be thirty days. This situation also conforms to the twelve zodiacal signs and [their] having thirty degrees each. Since the best known of the planets and celestial bodies are the two luminaries, most nations