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stationary individual from the west; the other travels toward the west and returns to the stationary individual from the east. All three have been counting the days. The easterner, for example, said: “Today is the hundredth day since we have left the stationary individual.” The westerner said: “No, it has been a hundred and two days.” The stationary individual said: “No, it has been a hundred and one days.” Or the easterner said: “Today is Saturday;” the westerner said: “It is Monday;” the stationary individual said: “It is Sunday.” None of them has made a mistake in counting. How can this be?

[10] Conceptually, this is [actually] true, since for the one who goes toward the east and returns from the west, one celestial revolution will be subtracted from his course because every day the Sun rises earlier for him, and so his nychthemeron gets shorter than those of the stationary individual; and during a [complete] revolution those deficits add up to a day. The Sun rises later every day for the one who sets out toward the west and returns from the east, and his nychthemeron gets longer than those of the stationary individual; those increments, distributed over the nychthemeron, add up to a revolution, which is one day. The stationary individual has his own standard [celestial] revolutions. These are the states of affairs of the locations of the Earth, which are dependent on the celestial conditions that have been mentioned—God the Almighty is all-knowing.

## CHAPTER SIX

On the Co-ascension of the Zodiacal Signs, Which Is the  
Rising of the Parts of the Equinoctial with the Parts of the  
Zodiacal [Orb]

[1] For every location, the rising of the parts of the zodiacal [orb] with the parts of the equinoctial is different from any other location because of the declination, as has been mentioned previously. The

amount of the equinoctial that rises with a given amount of the zodiacal orb is the co-ascension of those parts. In localities that are on the equator and hence have no latitude, the horizon of that locality is one of the declination circles, since it passes through the two poles of the equinoctial. Those localities are called straight horizons, and the co-ascension of those localities is called the co-ascension of the right orb, the co-ascension of the erect sphere, and the co-ascension of the equator.

[2] When an equinox point is on the horizon, the other [equinox] point is on the horizon in the other direction, and the two solstices are on the meridian circle. Thereafter, when an arc of the zodiacal orb rises, the arc on the equinoctial that rises is less than that, because the equinoctial intersects with the horizon at right angles, and the zodiacal orb at acute and obtuse angles. This situation continues until the equinox point reaches the meridian and the solstice point is on the horizon, at which time a quarter of each of these two equators will have risen completely and both circles intersect with the horizon at right angles. Thereafter, equal arcs of the zodiacal orb rise with unequal arcs of the equinoctial, as in the first quarter but in reverse order. For example, parts of Cancer remain in the sequence [of the signs], while parts of Gemini are in counter-sequence. [This continues] until another quarter rises and the equinox point reaches the horizon. All co-ascensions are as has been described for these two quarters.

[3] The co-ascensions of any four arcs equidistant from the two equinox points, such as the first ten degrees of Aries and the first ten degrees of Libra and the last ten degrees of Pisces and the last ten degrees of Virgo, are equal, as the co-ascensions of any four arcs equidistant from the two solstice points are equal. The initial point of the co-ascensions is taken to be the vernal equinox, [although] some [place it] at the beginning of Capricorn for another purpose, which shows up in practical [applications]. Since the passage of the parts of the zodiacal

[orb] across the meridian circle is similar inasmuch as the meridian circle is one of the declination circles, indeed being one of the horizons at the equator, in all localities the co-ascensions for the equator are used.

[4] As for the co-ascensions of the oblique horizons, which are those that have latitude and in which the turning [of the orb] is slanted, when the point of intersection is on the horizon, the solstice point is on the meridian. Then the point of intersection rises. If the arc of the zodiacal orb that rises is northerly, in northern habitations what rises from the equinoctial is less than that [arc], because the angles between the zodiacal orb and the horizon are more acute than angles between the equinoctial and the horizon; in southern habitations, it is the opposite. If that arc is southerly, in northern habitations what rises from the equinoctial with that [arc] is more than it; in southern habitations, it is the opposite.

[5] There are different rules for the quarters in these regions, because at the time a quarter of the zodiacal orb rises, not a full quarter of the equinoctial will have risen, if the arc of the zodiacal orb is in the direction of the local latitude; or, if it is in the opposite direction, more than a quarter will have risen. Nonetheless, the rules for the two halves of the zodiacal orb are the same, except that in one half it is in the sequence [of the zodiacal signs] and in the other it is in counter-sequence. Therefore, any two arcs equidistant from the equinox point have the same co-ascensions, and the co-descension of any sign is equal to the co-ascension of that sign's facing counterpart, because with [the rise of] any [zodiacal] sign, one will set. Therefore, the co-ascensions of the [zodiacal] signs in the north are exactly the same as the co-descensions of the [zodiacal] signs in the south, and the co-descensions of the [zodiacal] signs in the north are the co-ascensions of the [zodiacal] signs in the south.

[6] When the local latitude is equal to the complement of the obliquity, one half of the orb whose midpoint is the equinox point will rise in one stroke and there will be no co-ascension for it; in the other half, the entire equinoctial will rise. In localities whose latitude is greater than the complement of the obliquity, any two arcs that are permanently visible or permanently invisible will have no co-ascensions, but otherwise any two arcs will have co-ascensions, one in reverse order and the other in regular order. When the local latitude reaches the limit, the rotation of the orb being in a spinning manner, the co-ascensions will become completely baseless, since there will be no rising nor setting, and the equinoctial and the horizon will coincide.

[7] Some call the parts of the equinoctial “degrees,” and some “units of time,” because time is determined by the amount of its motion. This has been a description of the co-ascensions of the [zodiacal] signs—God is all-knowing.

## CHAPTER SEVEN

### On Determining the Equation of Daylight and the Ortive Amplitude in the Localities

[1] The arc on the horizon between the ascension of any part [of the zodiacal orb] and the ascension of the equinoctial is called the ortive amplitude of that part. It is obvious that the maximum ortive amplitude at the equator is equal to the obliquity. In other localities, the ortive amplitude increases with the local latitude, until the local latitude reaches the complement of the obliquity, and the ortive amplitude of a quarter of the orb is attained. The ortive amplitude of every quarter of the orb is equal to that of another quarter, one in the sequence [of the zodiacal signs] and the other in counter-sequence. The ortive amplitude of the two northern quarters is like that of the two southern quarters. [Also,] the ortive amplitude of any part is like the