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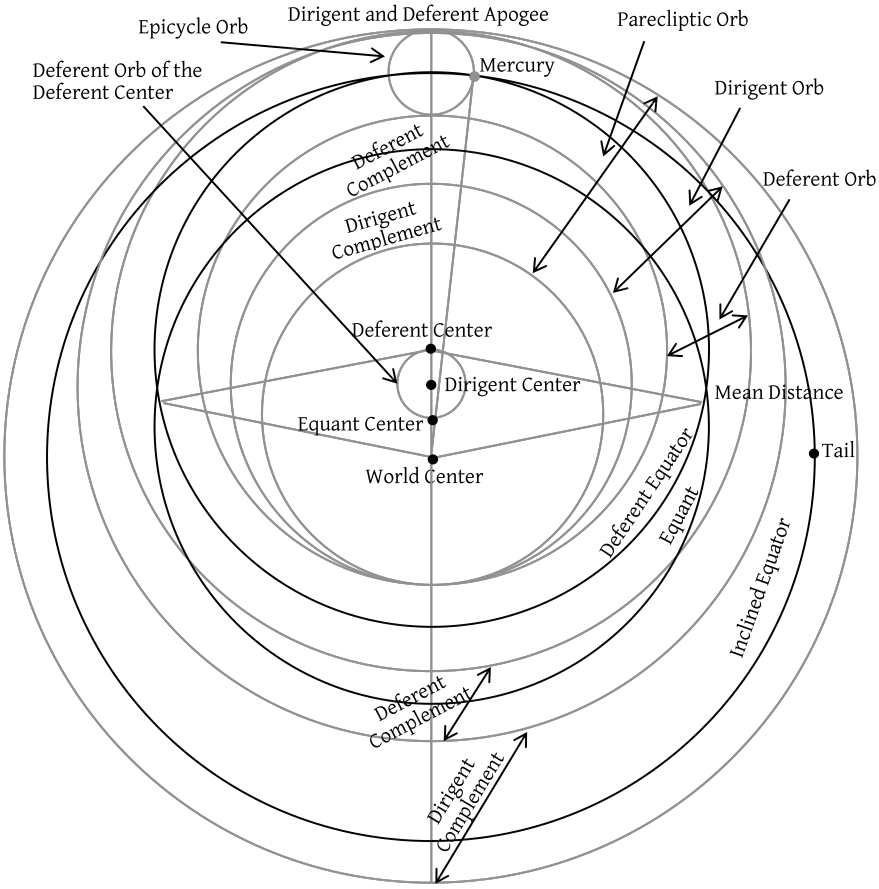


[Figure 2]

[22] Those parts that have been illustrated in black are the parts practitioners of this discipline are satisfied with. This much is sufficient for this chapter—God is all-knowing.

CHAPTER EIGHT

On the Latitudes of the Six Planets

[1] Turning to the latitudes of the planets: from what we have previously stated, namely that the center of the body of the Sun is always on the equator of the eccentric orb and that the equator of the eccentric orb is in the plane of the parecliptic equator whose plane is part of

the plane of the zodiacal orb, it is obvious from this that the Sun is always in the plane of the zodiacal orb and thus has no latitude. For this reason, the equator of the zodiacal orb is also called the solar circuit.

[2] As for the Moon, we have stated that it is on the equator of the epicycle orb, and the equator of the epicycle orb is in the plane of the eccentric orb's equator; the eccentric equator is in the plane of the equator of the inclined orb. Therefore, the Moon is always in the plane of the equator of the inclined orb, and the inclined equator is its circuit. Because the inclined equator intersects the parecliptic equator at two places, [i.e.,] the head and the tail, therefore when the Moon reaches the point of intersection it will be on the parecliptic equator and will have no latitude. In all other positions it will have latitude. While it is between the head and the tail, its latitude is northerly because the head is the northern crossing point; in the other half, it is southerly because the tail is the southern crossing point.

[3] When the point of intersection, i.e., the head, is taken as the starting point of the motion of the Moon's true position, which is a motion combined of the sum of the true motion of the Moon and the motion of the head, the latitude occurs due to this motion, and it is called the argument of latitude. When this argument of latitude is less than half a revolution, the latitude is northerly; when it is more, southerly. When a circle is imagined that passes through the four poles of the inclined and parecliptic [orbs], each half of the inclined orb is divided into two halves as well. Then, the first quarter, which the Moon enters as it passes the head, is the ascending latitude in the north and additive; in the second quarter, it is descending and subtractive; in the third quarter, which is after it passes the tail, it is descending in the south and additive; and in the fourth quarter, it is ascending and subtractive. This ascension and descension has been so established because the north[ern side of the zodiacal orb], from the viewpoint of the people of northern habitations, is of greater elevation than the south[ern

side]. The maximum latitude of the Moon in both directions is equal to the maximum declination of the inclined [orb] from the parecliptic, which is 5 degrees. Other than this, the Moon has no latitude.

[4] As for the upper planets, i.e., Saturn, Jupiter and Mars, each one has two latitudes. One is due to the declination of the inclined orb from the parecliptic orb, as has been said about the Moon. Since the center of the epicycle is on the equator of the eccentric, and the equator of the eccentric is in the plane of the inclined orb, this latitude is the latitude of the center of the epicycle orb. When the center is at either one of the two points of intersection of the parecliptic and inclined [equators], the latitude is zero. In other positions there is a latitude. The maximum latitude in both directions is: 2;30 for Saturn; 1;30 for Jupiter; and 1;0 for Mars. The circumstances of this latitude are exactly as has been said for the Moon.

[5] The second latitude is due to the apex and perigee of the epicycle orb, because the apex and perigee for these planets, or rather the plane of the equator of their epicycle orbs, does not remain fixed in the plane of the inclined equator, but rather the diameter that passes through the apex and perigee intersects the plane of the inclined orb. The inclination of the apex is always toward [one] side of the zodiacal orb, and the inclination of the perigee is toward the other side. Every time the epicycle center reaches one of two points, either the head or tail of that planet, this declination is nullified, and the plane of the epicycle equator coincides with the plane of the inclined orb. As long as the center of the epicycle is in the northern side, the inclination of the apogee is toward the south and the inclination of the perigee is toward the north. The maximum of each one [occurs] when the first latitude is at its maximum and [its] decrease [occurs] with the decrease [of the first latitude] simultaneously. When the center of the epicycle is toward the south, the inclination of the apex is toward the north and the inclination of the perigee is toward the south. The maximum

is like the maximum of the first latitude and is additive or subtractive to that latitude.

[6] The maximum of this latitude for each of the apex and the perigee, when the center of the epicycle is in the north, is: Saturn, 0;26, 0;32; Jupiter, 0;23, 0;34; Mars, 0;52, 3;21. In the south, they are: Saturn, 0;28, 0;35; Jupiter, 0;25, 0;38; Mars, 0;56, 6;07. When the [first and second] latitudes are combined, the inclination of Saturn's apex and perigee in the north is 2;4, 3;2 and in the south 2;2, 3;5; the inclination of Jupiter's apex and perigee in the north is 1;6, 2;4 and in the south 1;5, 2;8; the inclination of Mars's apex and perigee is 0;8, 4;21 in the north and 0;4, 7;7 in the south.

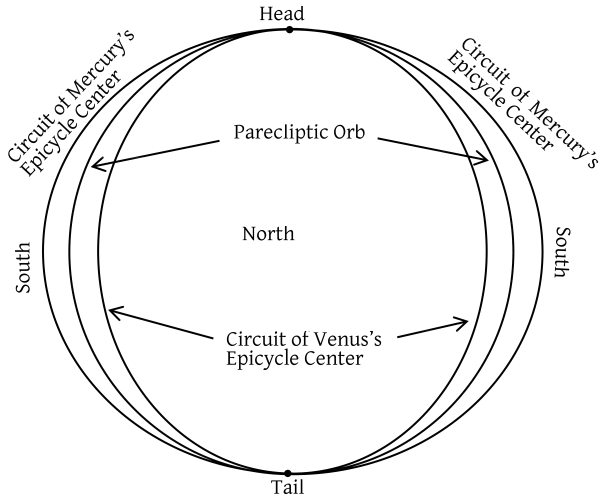
[7] In these planets, the diameter of the epicycle orb that passes through the two mean distances is always in the plane of the inclined orb. Since this latitude is due to a motion that is in the plane of the epicycle equator, it must assuredly have a mover. Abū ʿAlī ibn al-Haytham, who was one of the great scholars in the mathematical sciences, wrote a treatise on this matter and transformed each one of the epicycle orbs of these planets into three orbs enclosing one another: one is moved by the proper motion; another moves its equator in such a way that the diameter of the apex and perigee are inclined to the north and south; and the third is an orb that moves those two orbs in the counter-sequence so that the disruption caused by the motion of the second orb in the position of the first orb is eliminated.

[8] Yet even with this determination, this anomaly is not regulated, and in addition several other distortions come into being; but this is not the place to explain them. The author of the *Muntahá al-idrāk* says that the endpoint of the diameter that passes through the apex and perigee moves on the circumference of a small eccentric circle whose relation to another small circle is exactly that of the planet's eccentric to its inclined [orb]. But from what he says no defect that must be

removed is eliminated. Abū ʿAlī Haytham's treatise is still much nearer to the truth than this, and these words have been quoted exactly from Abū ʿAlī's and others' books.

[9] As for Venus and Mercury, both planets have three latitudes each. The first is the latitude from the inclination of the inclined orb from the parecliptic. This latitude is always northerly for Venus and southerly for Mercury. Practitioners of this discipline have said that the plane of the inclined orb intersects the plane of the parecliptic orb, as we have said; however, its inclination from that plane is not fixed. Rather, when the maximum has been reached, the plane of the inclined turns around and heads toward the plane of the parecliptic until both planes coincide. Then the inclined [orb] crosses the parecliptic [orb], and the half that was northerly becomes southerly and the half that was southerly becomes northerly, and so on until it reaches the maximum [inclination]; then it turns around, and so on and so forth. Then, when the center of the epicycle reaches the point of the head, the inclined plane will then coincide with the parecliptic plane. When the coincidence ceases to be and there is an inclination, the epicycle center of Venus is northerly and the epicycle center of Mercury is southerly. When each of these two centers reaches its maximum inclination, i.e., the midpoint between the two nodes, the inclination between the two planes attains its maximum. Thereafter the epicycle center heads toward the tail and the inclined plane reverts until, when the epicycle center reaches the tail, both planes coincide. Then, when one plane moves away from the other, the half that was northerly at that moment becomes southerly, and the half that was southerly becomes northerly. For Venus, the shift will have come in the half that was originally southerly but is now northerly, while for Mercury the shift will have come in the half that was originally northerly but is now southerly. Thus, Venus's epicycle center is always to the north of the parecliptic except at the two points of intersection at which it is

then on the parecliptic, and Mercury's epicycle center is always south of the parecliptic except at the two points of intersection at which it is then on the parecliptic. Since the eccentric is in the inclined plane, the apogee is northerly for half a revolution and southerly for half a revolution. This is an illustration of the circuit of the epicycle center for these two planets in relation to the parecliptic:



[Figure 1]

[10] The maximum latitude of Venus on either side [of the zodiacal equator] is $\frac{1}{6}$ of a degree; the maximum latitude of Mercury $\frac{1}{2}+\frac{1}{4}$ [of a degree]. Another orb must be posited for this motion, which the ancients did not do. This has been also indicated by Abū ʿAlī al-Haytham in the aforementioned treatise, but this is not the place to discuss that tract [of Ibn al-Haytham].

[11] The second latitude is the latitude of the diameter passing through the apex and perigee. When the center of the planet is at the midpoint of the two nodes, i.e., the head and tail, this latitude is zero on both sides [of the zodiacal equator], and the diameter that passes through the apex and perigee is in the plane of the inclined orb. When [the center of the planet] is at the head or tail, the inclination of the

diameter is at its maximum. An explanation of this is that when the epicycle center is at the midpoint between the head and the tail, which position is Venus's apogee and [the point] opposite Mercury's apogee, the apex begins to incline toward the north and the perigee begins inclining toward the south. Then, when it reaches the point of the tail, the apex is at its maximum northerly inclination and the perigee is at its maximum southerly inclination. When it reaches the midpoint between the tail and the head, the inclination disappears. After that, the apex begins its inclination toward the south and the perigee toward the north until, when it reaches the head, it will have attained its maximum. The maximum inclination for Venus's apex toward the north and south is 1;2, and the maximum inclination of its perigee in both directions is 6;20. The maximum inclination of Mercury's apex in both directions is 1;45, and the maximum inclination of its perigee in both directions is 4;4.

[12] As for the third latitude, which is called the slant, winding, or slope, this is the inclination of the diameter that passes through the two mean distances. The one-half of this diameter that is eastern and at which the planet appears before the Sun rises, is called the matutinal distance. The other half that is western and at which the planet appears after the Sun sets is called the vespertine distance. This inclination reaches its maximum when the epicycle center passes away from the head and is between the head and tail or between the tail and head, like the first latitude. When the center is at one of the two points of intersection, this inclination is zero. Then, when the epicycle center passes away from the head, the eastern side heads toward the north, and the western side toward the south, until when the center reaches the midpoint of the two nodes—i.e., the apogee for Venus and the [point] opposite the apogee for Mercury—this latitude will have reached its maximum. From there it keeps decreasing until it reaches the tail where the diameter is in the inclined plane. After passing away

from the tail, the eastern side heads southward and the western side northward, until it reaches its maximum at the midpoint, which is [the point] opposite Venus's apogee and Mercury's apogee, and then once again begins to decrease. The maximum of this latitude for Venus in both directions is 2;30, and for Mercury it is 2;15 in the apogean half and 2;45 in the perigean half. This latitude is the reciprocal of the second latitude, i.e., when that latitude is at its maximum, this latitude is zero, and when this latitude is zero, this latitude reaches its maximum.

[13] Abū ʿAlī Haytham has posited for each of Venus and Mercury five epicycle orbs, enclosing one another: one for the proper motion; the second for the inclination of the diameter of the apex and perigee; the third for maintaining the position that would otherwise be displaced due to [the second orb]; the fourth for the slant; and the fifth for maintaining the position that would otherwise be displaced due to the [fourth orb's] motion. This is [our] words concerning the latitudes of the six planets—God is all-knowing of the Truth.

CHAPTER NINE
An Exposition of the Planetary Sectors

[1] Each of the eccentric orbs and orbs of epicycles mentioned previously can be divided into four parts, each part of which is called a sector. Practitioners of this science disagree over where to place the beginning of the second and fourth sectors, but there is no dispute over the beginning points of the first and third sectors, which are the apogee and perigee or the apex and perigee.

[2] One group says that the initial points of the second and fourth sectors are the two points of mean distance, just as the beginning points of the first and third sectors are the points of the farthest and nearest [distances]. This being so, then a diameter must be assumed in the eccentric orb that passes through the apogee and perigee, and a line