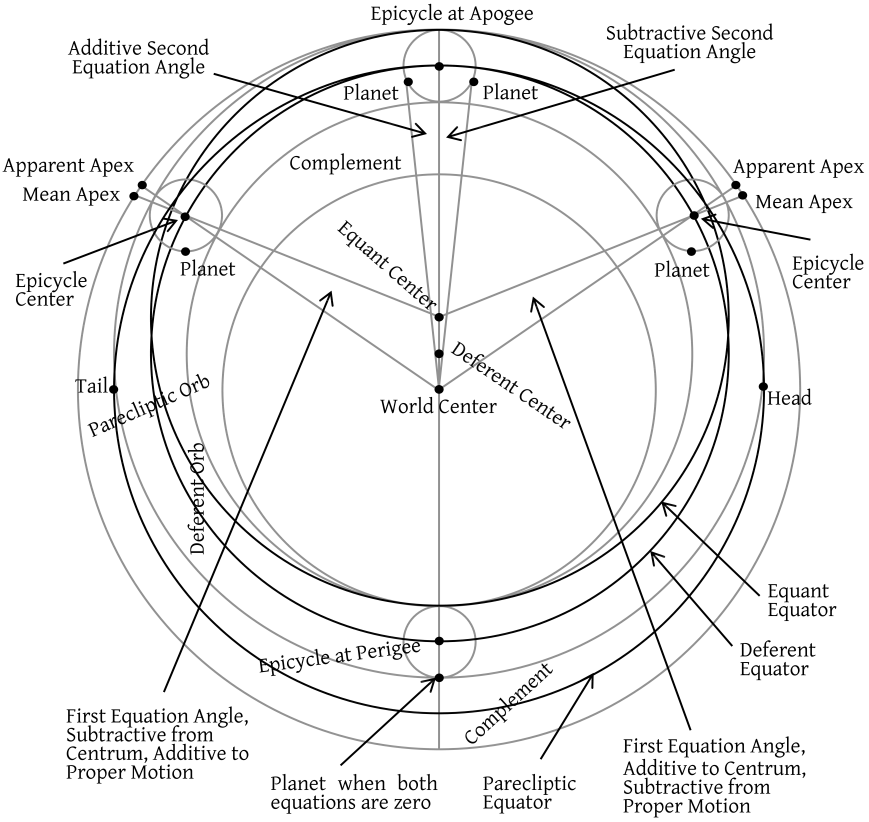
How to cite this document:

Naṣīr al-Dīn al-Ṭūsī. *al-Risāla al-Muʿīniyya*, book 2, chapter 7. In F. Jamil Ragep, Fateme Savadi, Sajjad Nikfahm-Khubravan. *al-Risāla al-Muʿīniyya (al-Risāla al-Mughniya) and its Supplement*. Vol. II, *English Translation* (Tehran: Mirath Maktoob), 69–76.



[Figure 1]

[19] The black circles represent the ones that practitioners of this discipline posit. The latitudes of the planets will be explained later. This is the configuration of the orbs of these planets—God is all-knowing.

CHAPTER SEVEN

An Exposition of the Orbs and Longitudinal Motions of Mercury

[1] The various situations of Mercury are as have been stated for Venus, except that Mercury's elongation from the Sun is never more than about 27 degrees, and that Venus has a perigee opposite its apogee, while Mercury does not have a perigee opposite its apogee but

rather [another] apogee as well, though not with the same distance and slowness as the first apogee. [Mercury's] perigees are almost at the two trines with respect to the apogee. The speed of [Mercury's] apogee is the same as the speed of the fixed stars.

[2] Then, to regulate these divergences, Mercury needs four solid orbs, three circles, and four uniform motions. The first orb is an orb whose center is the center of the World, and whose equator is in the plane of the zodiacal equator, whose convexity is contiguous with Venus's concavity, and whose concavity is contiguous with the Moon's convexity. This orb is called the parecliptic.

[3] The second orb is an orb whose center is eccentric to the center of the World, and it is situated in the thickness of the parecliptic orb as is the case of the eccentric [orb] in the other planets, and is tangent to both surfaces of the parecliptic at two points, one being the apogee and the other perigee, as we have said. The plane of its equator is outside the plane of the parecliptic equator. This orb is called the dirigent orb.

[4] The third orb is another orb whose center is eccentric to both the parecliptic's center and the dirigent's center. It is located in the thickness of the dirigent orb, and also is tangent to both surfaces of the dirigent orb at two points directly opposite one another, one being the apogee and the other the perigee. Its equator is in the plane of the dirigent equator; the relation of this orb to the dirigent orb is like that of the eccentric orb to the parecliptic orb in the other planets. This orb is called the deferent orb.

[5] The fourth orb is the epicycle orb, embedded in the thickness of the deferent orb so that its surface is tangent to both of the [deferent's] surfaces at two points, as in the other planets. The planet Mercury is embedded therein as with the other planets.

[6] As for the motions, the first motion is the motion of the parecliptic with the speed of the fixed stars. Because this motion appears in the

apogee, it is called motion of apogee.

[7] Second is the motion of the dirigent orb in counter-sequence, like the mean motion of the Sun; and by this motion all the rest of the orbs are moved. This motion is perceived in the apogee and perigee of the eccentric deferent [orb], and this motion is called the motion of the deferent apogee. Since through this motion the center of the deferent has a circuit around the dirigent center, this circuit is called the deferent orb of the deferent center, and this motion is called by the same name.

[8] The third motion is the motion of the deferent orb in the sequence of the zodiacal signs, equal to twice the Sun's mean speed. Since this motion is perceived in the epicycle center, this motion is called the motion of the center.

[9] The fourth motion is the motion of the epicycle orb in the same direction as the orbs of the epicycles of the other planets; it is 3;6 daily and this is called the proper motion.

[10] If a circle is conceived on the [exterior] surface of the parecliptic orb in whose plane the deferent equator lies, that circle is called the inclined orb. This [circle] necessarily intersects the parecliptic equator in two places that are called the head and the tail, and the motion of the fixed stars can be perceived in them.

[11] Then, when the center of the epicycle is at the deferent apogee, the deferent apogee will be coincident with the apogee point of the dirigent, and both apogees will be coincident at one common point on the upper surface of the parecliptic orb; the center of the epicycle will be at the farthest distance from the center of the World. Thereafter the deferent apogee moves in counter-sequence by the amount of the mean motion of the Sun, and the center of the epicycle moves in sequence by double the amount of the Sun's mean. By this [same] amount is the center elongated from the deferent apogee, and it is

elongated from the dirigent apogee by the amount of the Sun's mean. This is the amount of the motion of Mercury's center that is used in the *zīj*es. When the starting point is assumed to be the beginning of Aries, [the motion] is compounded of the motion of the dirigent apogee and this above-mentioned motion, whence it is called the mean motion [of Mercury].

[12] Thus, the dirigent apogee is always between the deferent apogee and the epicycle center, such that when each has traversed a quarter [revolution] of the dirigent orb, and both reach quadrature with respect to the dirigent apogee, the epicycle center will have reached the perigee of the deferent orb, which is directly opposite the apogee. When each has traversed another quarter [revolution] of the [dirigent] orb, the deferent apogee and the epicycle center will again meet opposite the dirigent apogee. Thus, the deferent apogee and dirigent perigee will meet, and the epicycle orb will be at the farthest distance from the center of the World—other than its first farthest distance. Then they move apart again, the deferent apogee going into the first half, and the epicycle center into the second half. At quadrature with respect to the dirigent apogee they are opposite once again, and the epicycle center reaches the deferent perigee. From there they are separated, and the two meet at the dirigent apogee. Since the second apogee—which is opposite the dirigent apogee—is nearer to the center of the World than the first apogee—which is compounded of both apogees—there is no true perigee—i.e., maximum proximity to the center of the World—at the two quadratures with respect to the dirigent apogee, as was said for the Moon. Rather, the true perigee in both directions is where it is compounded of both perigees [of the dirigent and deferent orbs], and in any case it is nearer to the opposition [point] of the dirigent apogee. Thus, the two perigees are almost at the trines with respect to the dirigent apogee, and at the sextiles with respect to the [point] opposite [the apogee].

[13] Because of the epicycle orb and its motion, retrograde and direct motions are produced; and because of the eccentric and its motion, the difference between one retrogradation and [another] retrogradation, and between one direct motion and another direct motion, are produced. On account of the dirigent orb and its motion, the epicycle center reaches the apogee twice per revolution, provided that one apogee is higher than the other, and [the epicycle center also reaches] the perigee twice per revolution provided that two perigees are equal; for if the motion were not in the counter-sequence [of the zodiacal signs], the apogee would not meet the center in its circuit, as we have said for the Moon. If this orb were not eccentric, both apogees would be equal in distance, as it is for the Moon. Because of the parecliptic orb, the dirigent apogee moves in sequence with the speed of the fixed stars. By these orbs and motions the above-mentioned anomalies are ordered.

[14] Mercury also has three anomalies. First is the anomaly from the radius of the epicycle, i.e., the second equation, which is [the angle between] two lines extending from the center of the World to the center of the epicycle and to the body of the planet. Its maximum is in the amount of the radius of the epicycle, and this amount is 22;30.

[15] Second is the anomaly due to the epicycle being at different distances, i.e., the inequality of farthest and nearest distances.

[16] Third is the anomaly due to the alignment of the epicycle's diameter with a point that is other than the above-mentioned center points. This is the first equation.

[17] The state of being additive or subtractive in these anomalies is just the same as being additive or subtractive in the other planets.

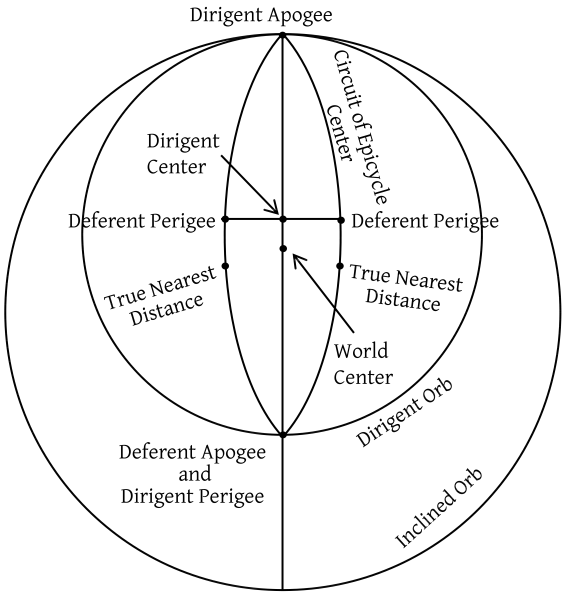
[18] The point with which the mean apex and perigee are aligned for this planet lies between the dirigent center and the center of the World. Thus, when the deferent apogee and dirigent apogee coincide,

these four centers are along one line: first is the center of the World; above it is the above-mentioned point which is the center of the equant circle; above it is the dirigent center; and above it is the deferent center. The distance between each two of these centers is 3;0, assuming the radius of the eccentric to be 60. The equant circle for this planet is, as in the other planets, the same size as the deferent equator; its position, however, is toward perigee, unlike the other planets. Since the distance of the deferent center from the dirigent center is equal to the distance of the equant center from it, both centers are thus on the circumference of a small circle [known as] the deferent of the deferent center. Since the deferent center moves around this circle, it reaches the equant center once in every revolution, [whereupon] the equant and deferent coincide, and afterwards are separated once again. They coincide when the epicycle center is at the closer apogee. Practitioners of this discipline are content to set forth circles and put the deferent orb of the deferent center in place of the dirigent orb. The other circles are in the manner that was mentioned previously.

[19] Therefore, from this discussion [the following] has been established: four solid orbs, namely, the parecliptic orb, the dirigent orb, the deferent orb, and the epicycle orb; three circles, namely, the inclined orb, the equant orb, and the deferent orb of the deferent center; four simple uniform motions, namely, the motion of the apogee, the motion of the center, the motion of the dirigent, and the proper motion; two uniform compound motions, namely, the mean motion and the motion of the center used [in the *zījes*];[1] and three compound motions, namely, the adjusted motion of the center, the adjusted proper motion, and the motion of the true position. The meanings of the names are the same as has been previously stated.

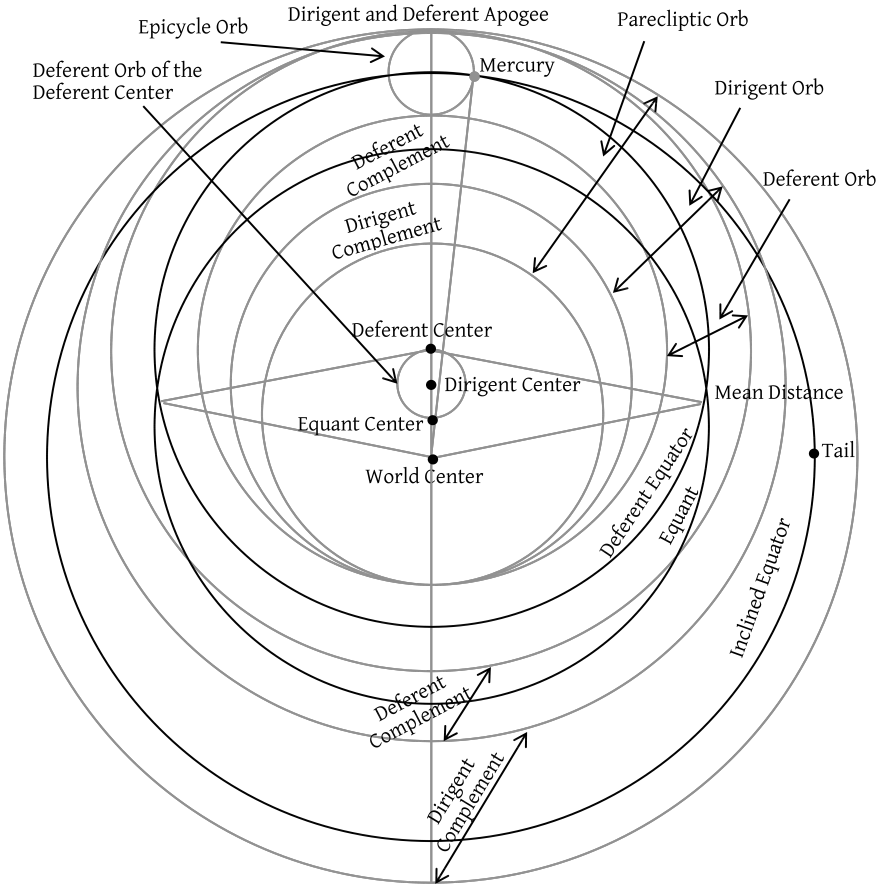
1. For "the motion of the center used [in the *zījes*]," see paragraph 11 of this chapter.

[20] Between the head and the apogee of Mercury is 270 degrees. Since this planet has two eccentric orbs, there are four complementary solids: two from the dirigent orb and two from the parecliptic orb. Here is an illustration of the circuit of the center of the epicycle in relation to the centers of the World:



[Figure 1]

[21] The doubt of which we spoke concerning the equant and the eccentric applies to these orbs as well, and the solution also is as in the other [planets]. The illustration of the orbs of Mercury is this:



[Figure 2]

[22] Those parts that have been illustrated in black are the parts practitioners of this discipline are satisfied with. This much is sufficient for this chapter—God is all-knowing.

CHAPTER EIGHT

On the Latitudes of the Six Planets

[1] Turning to the latitudes of the planets: from what we have previously stated, namely that the center of the body of the Sun is always on the equator of the eccentric orb and that the equator of the eccentric orb is in the plane of the parecliptic equator whose plane is part of