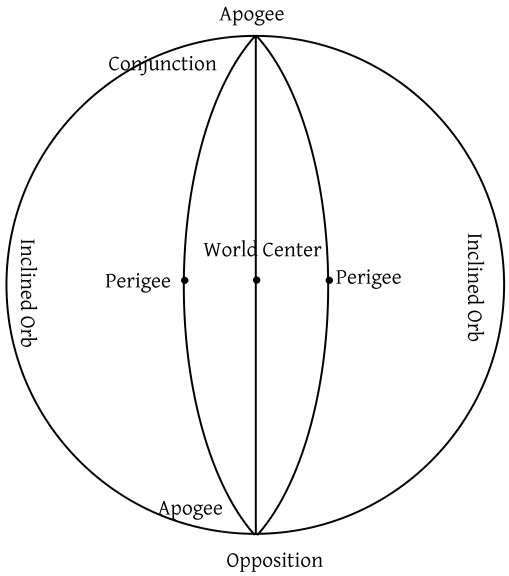
How to cite this document:

Naṣīr al-Dīn al-Ṭūsī. *al-Risāla al-Muʿīniyya*, book 2, chapter 6. In F. Jamil Ragep, Fateme Savadi, Sajjad Nikfahm-Khubravan. *al-Risāla al-Muʿīniyya (al-Risāla al-Mughniya) and its Supplement*. Vol. II, *English Translation* (Tehran: Mirath Maktoob), 61–69.



[Figure 2]

[28] Other situations concerning the Moon, such as latitude, lunar eclipses, parallax, etc. will be treated later in their proper places—God willing.

CHAPTER SIX
An Exposition of the Orbs and Longitudinal Motions of the Upper Planets and Venus

[1] When the situation of the other planets is contemplated, [it will be seen that] Saturn, Jupiter, Mars, and Venus, all four, have direct and retrograde motion in common. The upper planets undergo combust at the midpoint of the direct motion period and are in opposition with the Sun at the midpoint of the retrogradation period; Venus undergoes combust in both situations. Venus's maximum distance from the Sun is not more than around 47 degrees. The direct motion of these planets is when they are distant from the Earth, and [their] retrogradation is when they are in proximity to the Earth. If one retrogradation is com-

pared with another retrogradation, and direct with direct, they will be found to be of varying periods. Whatever happens in a given part of the [zodiacal] orb in the shortest period will happen in its directly opposite part in the longest period, and that part of the [zodiacal] orb in which this is known to take place is displaced over a very long period of time, like the fixed stars. The circuits of these planets are not the circuit of the Sun but rather slant sometimes toward the north and sometimes toward the south.

[2] Therefore, through much contemplation, it became clear that if three solid orbs and three uniform motions are posited, [all of] these situations will fall into order in accordance with the mentioned principles. Each of these planets then has three orbs and three motions.

[3] As for the first orb, it is an orb whose center is the center of the World, its equator is in the plane of the zodiacal [orb], its convexity is contiguous with the concavity of the orb above it, and its concavity is contiguous with the convexity of the orb beneath it. As in Saturn, the convexity of this orb is contiguous with the concavity of the fixed stars, and its concavity is contiguous with the convexity of Jupiter's orb; and for Jupiter, its convexity is contiguous with Saturn's concavity, and its concavity is contiguous with Mars's convexity, and so on according to this pattern for Mars and Venus. This orb is called the parecliptic orb.

[4] As for the second orb, the orb is an eccentric, in the thickness of this [parecliptic] orb, as was said for the Sun, except that the plane of this orb's equator is inclined to the plane of the parecliptic equator. If a circle is conceived on the surface of the parecliptic orb in such a way that [this second orb's] equator be in the plane of that circle, that circle would intersect the parecliptic equator at two places, and that circle is called the inclined orb. Those two points [of intersection] are called the head and the tail, as was said concerning the Moon. And this orb is called the eccentric orb, as well as the deferent orb.

[5] As for the third orb, it is an epicycle orb, in the thickness of the deferent, as has been said for the Moon.

[6] As for the motions: the first motion is that of the parecliptic orb, which moves with the motion of the fixed stars. Since it is perceptible at the apogees and nodes, it is called the motion of the apogees. Since we have already said that the mover of this motion is the eighth orb, there is no need for positing this body [as the cause] for this motion, unless an eccentric orb is posited, [in which case] two complementary solids are certainly produced. The sum of these two complements with the eccentric is then an orb, since there is no empty place in the World, as is established in the principles of physics.

[7] The second motion is that of the deferent [orb]. It is 0;2 per day for Saturn; 0;5 for Jupiter; 0;31 for Mars; and 0;59 for Venus. Since this motion is perceptible in the epicycle center, it is called the motion of the center when the starting point of this motion is taken at the farthest distance; if the starting [point] of the motion is placed at the first of Aries, it is called mean motion.

[8] The third motion is that of the epicycle orb. It is 0;57 per day for Saturn, 0;54 for Jupiter, 0;28 for Mars, and 0;37 for Venus. Since this motion is perceptible in the bodies of the planets, it is called proper motion. The direction of these motions in these planets is opposite to what we said for the Moon, since for these planets it is in the sequence of the signs in the apical half but in the counter-sequence [of the zodiacal signs] in the perigean half. Due to this motion, the distance of the planets from the center of the World varies, as does faster and slower [motion] as well as direct and retrograde motion, because in the half where the motion conforms to the sequence of the zodiacal signs, the planet moves with direct motion, while in the other half where the motion is in the counter [sequence], the planet retrogrades. Retrogradation will be discussed in more detail later—God willing.

[9] Due to the motion of the epicycle around the deferent orb, a difference in ratio occurs between two retrogradations, or between two direct motions, or between two faster motions, or between two slower motions. This is because whenever the epicycle is at the apogee, a difference resulting in slower motion, which is due to the eccentricity of the deferent center—as was said for the Sun—is added to the difference that is due to the epicycle. And when the epicycle is at the perigee, that difference results in faster motion. Because the parecliptic moves with the motion of the fixed stars, the apogee and perigee, which are the midpoints of the periods of slower motion and faster motion [respectively], are displaced by the motion of the fixed stars. Inasmuch as the amount of proper motion of Saturn, Jupiter, and Mars is equal to the excess of solar mean motion over the motion of the centers of their epicycles, if we assume that the planet is at the apex and [hence] in combust, every day thereafter it will move away from the apogee by the amount of its proper motion, and this amount will be the distance between the centers of [their] epicycles and the Sun. Then, when each one of these two distances has gone half a revolution, the planet will have reached the epicyclic perigee and the Sun will reach opposition with the epicycle centers. When a full revolution has been made and the planet has reached the apex [once again], it will be in combust. The apex is the midpoint of the period of direct motion, and the perigee is the midpoint of the period of retrogradation. For this reason, these three planets always undergo combust when at the apex and at the midpoint of direct motion, and are in opposition with the Sun at the perigee and at the midpoint of retrogradation.

[10] Here is a question posed for the practitioners of this discipline. Why is it that Mars is closer to the Sun when it is in opposition to the Sun than it is when it is at combust? The answer is that since Mars's combust occurs at the apex, [the distance] between Mars and the Sun at the time of combust is the diameter of Mars's epicycle plus more,

due to the complement of its orb and the Sun's orb. And because its opposition to the Sun occurs at its epicycle's perigee, at that time [the distance] between it and the Sun is the diameter of the Sun's orb plus more, due to the complements. The diameter of Mars's epicycle orb is greater than the diameter of the Sun's orb; therefore, Mars is farther from the Sun during combust than during opposition.

[11] Since the motion of Venus's center conforms to the Sun's mean motion, the center of its epicycle is always aligned with the Sun; it therefore undergoes combust at the midpoint of its direct motion and the midpoint of retrogradation, i.e., at apex and perigee. Venus's distance from the Sun does not exceed the radius of its epicycle, which is around 47 degrees.

[12] Each of these planets has three anomalies: The first arises out of the two lines extending from the center of the World, [one] to the center of the epicycle and [the other] to the center of the planet's body. This is the independent, or second equation—as we mentioned for the Moon—except that [here] between apex and perigee it is additive and in the other half subtractive, contrary to the Moon, because the direction of the proper motions of these [planets] is opposite to that of the Moon's proper motion.

[13] The second anomaly comes about from the difference between [the amount of] the radius of these planets' epicycles at the farthest distance and the nearest distance. This is called the anomaly of the farthest and nearest distance. Its situation is close to what was described for the Moon, except in being additive or subtractive, since a difference occurs due to the way of calculation chosen by the author of the *Almagest*. This is because he calculated the second equation of the Moon assuming the epicycle at the farthest distance, for which reason the anomaly will be additive [as the second equation] increases, and subtractive as it decreases. He [also] calculated the second equation of

the planets at the mean distance, for which reason the anomaly is subtractive in the side of the farthest distance, when the second equation is additive, and when the second equation is subtractive, the anomaly is additive. In the side of the nearest distance the opposite occurs, as we mentioned for the Moon. This difference is due not to a variance in the configuration but to a variance in the [*Almagest*] author's way [of calculation].

[14] The third anomaly is that resulting from the alignment of the apex and perigee, for the diameter that passes through the apex and perigee in the epicycle orb is not aligned with the center of the deferent nor with the center of the World, but rather with a point whose distance from the center of the deferent, in the direction of the apogee, is equal to the distance of the deferent center from the center of the World. If one conceives of a line extending from that [above-mentioned alignment] point that reaches the epicycle orb, and rotates with it, it is called the dirigent line. From this line, and a line extending from the center of the World to the center of the epicycle, an angle arises at the center of the epicycle, which is called the angle of the first equation. In the amount of this angle there occurs a difference between the mean apex, which is aligned with the [alignment] point and the visible apogee, which is aligned with the center of the World. Since the starting point of the proper [motion] has been taken from the mean apex, then this equation must be added to the proper [motion] in the half where the epicycle center is between the apogee and perigee, and subtracted in the other half in order to adjust the proper motion and make the visible apex its starting point. Even though [the direction of] the proper motion of the planets is opposite to that of the Moon, nonetheless because the alignment point in the Moon is on the side of the perigee, while in the planets it is on the side of the apogee, being additive or subtractive for this equation is as for the Moon.

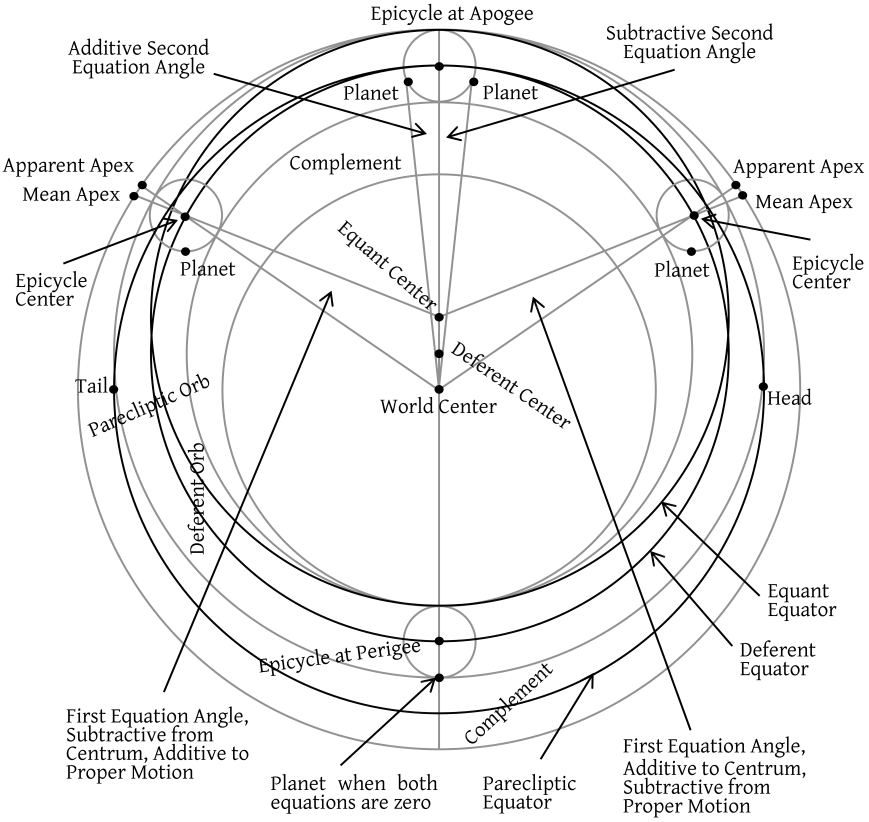
[15] The motion of the center of the epicycle for these planets is not uniform, neither with respect to the center of the World nor to the deferent center, but uniform with respect to the aforementioned [alignment] point. Then, a circle is conceived with the [same] size as the equator of the deferent with its center at the aforementioned [alignment] point; that circle is called the equant orb on which equal arcs will be cut in equal times. Thus, the difference of the motion of the center with respect to the center of the World is exactly brought about from the angle of the first equation—as was said in [the case of] the Sun. So in the half in which we add this equation to the proper [motion], it must be subtracted from the center; and in the half that it is subtracted from the proper [motion], it must be added to the center in order to adjust the center. This is not necessary for the Moon because the motion of the Moon's center is uniform around the center of the inclined orb.

[16] As for the doubt that occurred for the Moon, the exact same [doubt] applies to the motion of the epicyclic center on the equator of the deferent, with non-uniformity with respect to its center but with uniformity about another center eccentric to it. There follows one of the two aforementioned impossibilities: either there is uniformity about the deferent center so that the first equation becomes half of what it is in use; or else there is non-uniformity about the deferent center, so that the variations in the distance between the center of the epicycle and the center of the World become twice what it is known to be. The resolution of this doubt, about which none of the practitioners of this discipline has raised any objection and remains one of the mysteries of astronomy, will be explained at some other time—God willing.

[17] Practitioners of this discipline content themselves with circles, as was mentioned for the Moon. Now it is known that each of these planets has three solid orbs, two circles, and three simple uniform motions.

The orbs are: the parecliptic orb; the deferent orb; and the epicycle orb. The circles are: the inclined orb; and the equant orb. The motions are: the motion of the apogee; the motion of the center; the proper motion; and the fourth [motion], which is compounded of the motions of the apogee and the center and is the mean motion. If we consider the non-uniform, compound motions, the adjusted motion of the center, the adjusted proper motion, and the motion of the true position would be added.

[18] The distance between the eccentric center and the center of the World is: 3;25 for Saturn; 2;45 for Jupiter; 6;0 for Mars; and 1;2 for Venus—the radius of the eccentric being 60 degrees. Twice these amounts would be the distances between the equant center and the center of the World, which are used in the calculation of the first equation. The radius of the epicycle for Saturn is 6;30, for Jupiter 11;30, for Mars 39;30, and for Venus 43;10—the radius of the parecliptic being 60. These amounts are used in the calculation of the second equation. Between the head of a node and the apogee of Saturn is always 140 degrees; between the head and apogee of Jupiter 70 degrees; between the head and apogee of Mars 90 degrees; and between the head and apogee of Venus the same amount [90 degrees]. The terms used for them are close to what we have specified for the Moon, so we will not repeat them to avoid prolongation. The following is the illustration of the orbs of these planets:



[Figure 1]

[19] The black circles represent the ones that practitioners of this discipline posit. The latitudes of the planets will be explained later. This is the configuration of the orbs of these planets—God is all-knowing.

CHAPTER SEVEN

An Exposition of the Orbs and Longitudinal Motions of Mercury

[1] The various situations of Mercury are as have been stated for Venus, except that Mercury's elongation from the Sun is never more than about 27 degrees, and that Venus has a perigee opposite its apogee, while Mercury does not have a perigee opposite its apogee but