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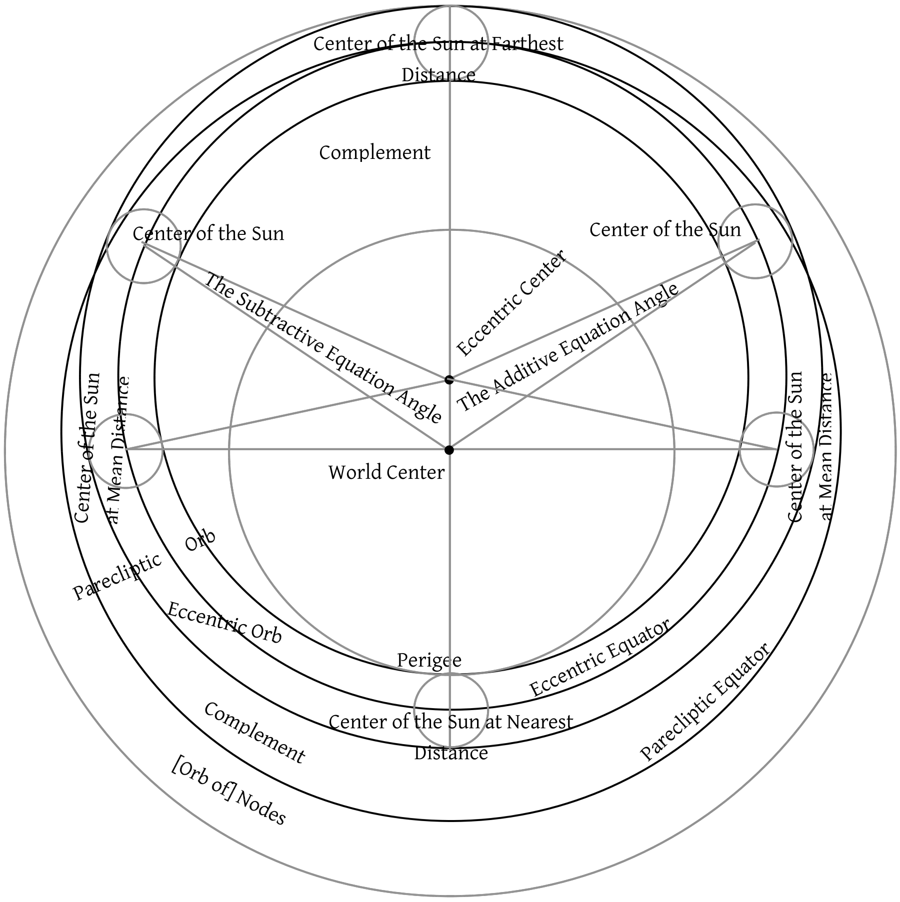


[Figure 1]

CHAPTER FIVE

An Exposition of the Orbs and Longitudinal Motions of the Moon

[1] When the situation of the Moon is considered, a faster and a slower motion will also be found in the course of its movement, and [also] an approach toward the Earth and a receding from it. [This] approach and receding of [the Moon] is determined by parallax. However, its situation in these positions is opposite to the situation of the Sun, since the Sun always has a slow speed when it is situated farther from the

Earth and has a fast speed when it is situated nearer. The Moon may sometimes be at the farthest distance with a fast speed, or again may sometimes be at the farthest distance with a slow speed. [Likewise] it may sometimes be at the nearest distance with a fast speed, or may be again at the nearest distance with a slow speed.

[2] Its farthest distance from the Earth is always at the time of conjunction and opposition, and its nearest distance is during the two quadratures with the Sun. Its circuit does not correspond to the Sun's, but rather is sometimes north of the Sun's circuit—i.e., of the zodiacal equator—and sometimes south. The two points of intersection between these two circuits move from east to west. Therefore, due to these irregularities, four solid orbs and four uniform motions were posited so that these matters could be ordered by compounding these bodies and motions.

[3] As for the orbs, the first orb is an orb whose center is the center of the World and whose two poles and equator are congruent with the poles and equator of the zodiacal orb and in the same plane. Its convexity is contiguous to the concavity of Mercury's orb, and its concavity is contiguous to the second of the Moon's orbs. This orb is called the parecliptic.

[4] The second orb is an orb whose center is also the center of the World, and whose equator is not in the plane of the zodiacal orb, but rather one half [of it] is north of that equator [of the zodiacal orb] and one half south of it, like what we have said in the case of the equinoctial and the zodiacal orb. Its two poles are on opposite sides with respect to the two poles of the parecliptic. Its convexity is tangent to the concavity of the first orb, and its concavity is tangent to the world of generation and corruption. This orb is called the inclined orb.

[5] The third orb is an eccentric orb in the thickness of the inclined orb, in the same way that the Sun's eccentric is in the thickness of its

parecliptic, i.e., its equator is in the plane of the inclined orb's equator, its convexity is tangent to the [latter's] convexity at a single point, and its concavity is tangent to the [latter's] concavity at a single point.

[6] The fourth orb is an epicycle orb, embedded in the thickness of the eccentric orb, as has been explained previously, such that its convexity is tangent to both surfaces of the eccentric orb at two points. The Moon's body is situated in the epicycle orb like a jewel in a ring, such that the convexity of its spherical body is tangent to the convexity of the epicycle orb at one common point.

[7] As for the motions: the first motion is the motion of the parecliptic orb in the counter-sequence of the zodiacal signs, 0;3 per day, and all the remaining orbs are carried with this motion. Since this motion is perceived at the intersection of the equator of the parecliptic and the inclined, it is called the motion of the nodes (*jawzahar*), because the two intersections are called the *jawzahar*. The parecliptic orb is called the *jawzahar* orb. The intersection of these two equators is conceived on the inclined orb, just as we have said concerning the intersection of the equinoctial and the zodiacal orb. Of these two intersections, the one that the Moon upon reaching it becomes northerly with respect to the Sun's circuit is called the northern crossing point or the head; and the other, which is directly opposite it, is called the southern crossing point, or tail. The maximum obliquity between these two equators in either direction is 5;0, and this is the maximum latitude of the Moon.

[8] The second motion is that of the inclined orb, which is also in the counter-sequence of the zodiacal signs, 11;9 per day. Because this motion is perceived at the apogee and perigee of the eccentric [orb], it is called the motion of the apogee and the farthest distance. The eccentric orb and epicycle orb move with this motion as well.

[9] The third motion is that of the eccentric orb moving in the sequence of the zodiacal signs, 24;23 per day. Because this motion is

perceptible at the epicycle center, it is called the motion of the center. The epicycle moves with this motion.

[10] The fourth motion is that of the epicycle orb, 13;4 per day along its circumference, such that in the upper half it is in the counter-sequence of the zodiacal signs and in the lower half it is in the sequence of the zodiacal signs. Since this motion is perceptible in the body of the Moon, it is called the proper motion.

[11] The farthest position of the epicycle orb with respect to the center of the World is the apex, and its closest position is the [epicyclic] perigee.

[12] The equator of the epicycle orb is always in the plane of the equator of the eccentric orb, and the equator of the eccentric orb is in the plane of the equator of the inclined orb. The eccentric orb is also called the deferent orb of the epicycle.

[13] Therefore, due to the motion of the epicycle orb and the displacement of the Moon's body with it, fastness and slowness are produced in the Moon's motion. Because in the upper half, where the motion of the epicycle is in the counter-sequence [of the zodiacal signs], the [combined] motion [which is] in the sequence [of the zodiacal signs] is slower by the amount of that motion subtracted from [the sequential motion]. In the lower half it is faster, because the two motions agree [in direction], in the amount of the addition of this motion to it. Because of the motion of the eccentric orb and the displacement of the epicycle by it, the Moon approaches and recedes from the Earth, so that it can be fast or slow whether far or near.

[14] Since the parecliptic orb moves the other orbs in the counter-sequence [of the zodiacal signs], the two nodes move in the counter-sequence. Since the Moon is in the plane of the inclined orb, and the plane of the inclined orb is slanted with respect to the parecliptic plane, the Moon gains latitude on the zodiacal orb to the north and

south. Therefore, its circuit is different from that of the Sun and intersects the Sun's circuit at two places. The center of the lunar epicycle at the time of conjunction and opposition is always at the deferent orb's apogee. Therefore, since the parecliptic moves 0;3 daily in the counter-sequence, and the inclined 11;9, also in the counter-sequence, the apogee moves away from its initial position by the sum of these two amounts: 11;12. And since the center of the epicycle moves 24;23 in the sequence, the apogee moves away from the center of the epicycle by this [same] amount. Therefore, the distance of the center of the epicycle from its initial position amounts to 13;11, and this is the mean motion of the Moon, since the displacement of the Moon in the signs appears to be this much.

[15] The Sun moves 0;59 [degrees per day] in the sequence from its initial position; [if] we subtract this amount from the mean motion [of the Moon], 12;12 remains, which is the distance of the center of the epicycle from the Sun. If we add this same amount to the distance of the apogee from its initial position, it becomes 12;12, which is the distance of the apogee from the Sun, and it is equal to the distance of the center of the epicycle from the Sun. Therefore, the Sun is always at the midpoint between the apogee and the center of the epicycle. The distance of the apogee from the center of the epicycle is twice the distance of each one from the Sun. For this reason, the motion of center of the epicycle is called the double elongation.

[16] In this manner, the apogee moves in the counter-sequence and the center of the epicycle moves in the sequence, so that when each one has completed half a circuit, they will be together at opposition of the Sun. Then, in opposition, the center of the epicycle is at apogee once more, and [the two] again pass away from each other until they reach each other in conjunction. At the quadratures, the apogee and the center of epicycle stand opposite one another. So, the center of the epicycle is always at apogee in conjunction and opposition, and at

perigee in quadratures.

[17] From these configurations, there result three anomalies for the Moon: the first [anomaly] is the difference that arises from two lines extending from the center of the World, one to the center of the epicycle and the other to the center of the body of the Moon. This difference is called the independent equation, and in some *zīj*es the second equation. This is on account of the proper [motion], and its maximum reaches the amount of the radius of the epicycle orb. The amount of the radius of the epicycle orb, assuming the radius of the inclined orb as 60, is 5;15. When the Moon is at epicyclic apex or perigee, the two above-mentioned lines coincide with each other and there is then no equation. Since its motion from apex is in the counter-sequence, during the time that the Moon is between apex and perigee, the equation is subtractive, and additive in the other half.

[18] The second anomaly arises from the approach of the epicycle orb toward and its receding from the center of the World. Accordingly, whenever the center of the epicycle is at the apogee of the deferent, the radius of the epicycle appears less than when at the deferent's perigee. This difference is called the anomaly of the farthest and nearest distance. This difference being additive and subtractive follows the independent equation's being additive and subtractive, since [this anomaly] is actually dependent upon it.

[19] The third anomaly results from the proper motion on account of the mean apex and perigee of the epicycle orb not being the apparent apex and perigee, which are [defined] with respect to the center of the World, i.e., the diameter of the epicycle that passes through the apex and perigee. It is not aligned with the center of the eccentric, nor with the center of the World, except when the center of the epicycle is at apogee or perigee of the deferent orb. However, it is always aligned with a point whose distance from the center of the World, in the direc-

tion of the perigee, is equal to the distance of the center of the deferent from the center of the World. The amount of distance between these two centers, assuming the radius of the deferent as 60, is 12;30. This [amount] is used to calculate this anomaly. By assuming the radius of the inclined [orb] as 60, it is 10;19; and this [amount] is used to find the distance of the Moon from the Earth.

[20] Then, when the center of the epicycle is at apogee or perigee, this third anomaly is imperceptible, because the diameter of the epicycle is aligned with the center of the World as well as the center of the eccentric, and prosneusis point. However, when it is between apogee and perigee, the end of a line extending from the prosneusis [i.e., alignment] point to the center of the epicycle and thence to the equator of the inclined [orb] is closer to the apogee than is the end of a line extending from the center of the World; [so] the apparent apex is farther from the apogee. Therefore, the distance between the two apices must be added to the proper motion, the initial position of which is taken from the mean apex, to obtain the adjusted proper motion. In the other half, it must be subtracted. This difference is called the equation of proper motion, or the first equation. The angle occurring from these two lines at the center of epicycle is called the angle of the first equation; similarly, the angle occurring from the two lines extending from the center of the World to the center of the epicycle and to the body of the Moon is called the angle of the second equation. The mean distance of the Moon in the apogee orb is where two lines extending from the center of the World and the center of the eccentric [to the center of epicycle] are equal—as we have said for the Sun.

[21] Thus, when four orbs and four motions are posited for the Moon, this irregularity will be resolved. The orbs: the parecliptic orb; the inclined orb; the deferent orb; and the epicycle orb. Instead of orbs, the practitioners of this discipline posit circles, which are the equators of these orbs, and call them by these [same] names—as has been said

for the Sun. Thus, the parecliptic and inclined are two intersecting circles with equal radii; and the deferent is an eccentric circle that passes through the center of the epicycle and is tangent to the inclined [orb]; and the epicycle circle is the equator of the epicycle orb. A circle is also produced by the circuit of the center of the eccentric around the center of the World, which is called the deferent of the center of the deferent orb.

[22] The motions are: the motion of the nodes; the motion of the farthest distance; the motion of the center, which is called the motion of the double elongation; and the proper motion. What arises from these motions is the mean motion. The compound non-uniform motions are the adjusted proper motion and the motion of the true position.

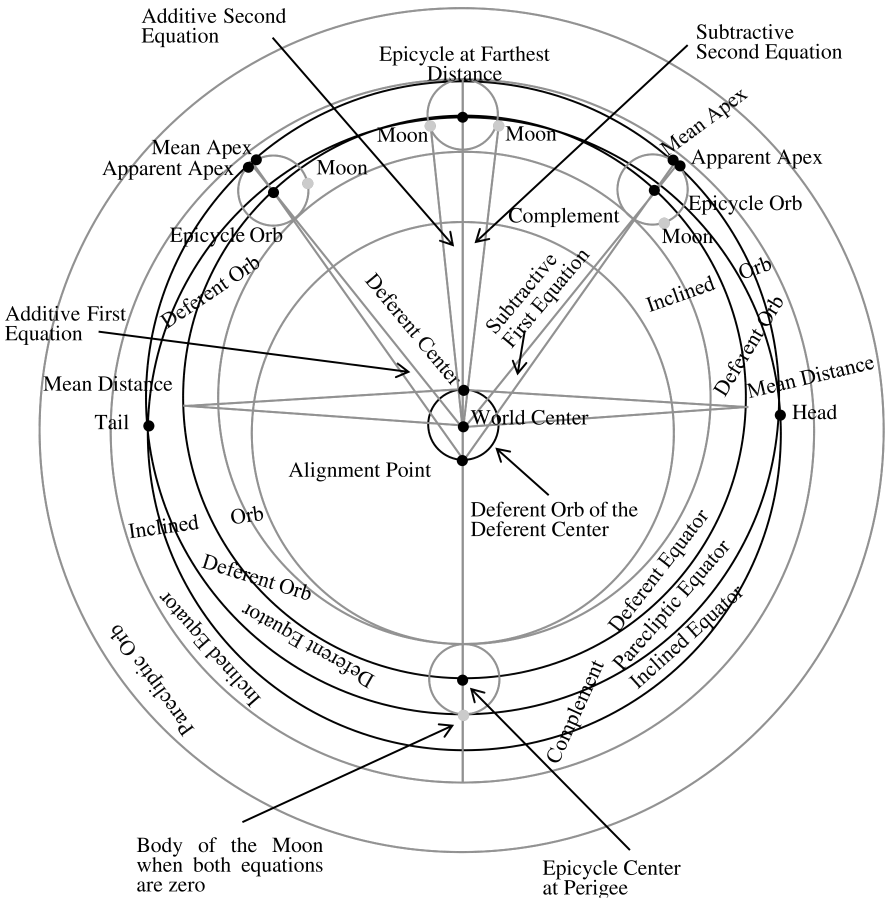
[23] Practitioners of the discipline have said that the motion of the fixed stars, which arises from the eighth orb, also pertains to the Moon but, because of [the Moon's] fast motions, it is not perceptible. This caveat they have made is utter nonsense, since, from the time of Battānī's observation, which is [fairly] close to us, the fixed stars have moved 5 degrees. If such a difference had befallen the Moon's motion, think what it would be like! If there were a 20-minute difference, for example, in the true position of the Moon, there would be such discrepancies in the lunar and solar eclipses, etc. that they could not be represented. [Now] the reason that the motion of the fixed stars is imperceptible [in the Moon] is what we have said, that two different [circular] motions in one sphere, around one equator and two given poles, are indistinguishable—rather, one motion can be perceived from the combination [of the two]. Since the motion of the nodes of the Moon's orbs is around the equator of the zodiacal orb and its poles, the motion of the fixed stars combined with that motion cannot be distinguished. Therefore, the motion of the nodes is in fact more than this perceived amount by the amount of the fixed stars' motion; thus, on account of the difference between these two motions, this amount, which has

been found by observation, has come to be perceived. This being so, there are five uniform motions.

[24] [However,] a doubt arises from the foregoing account. This [doubt] is that from the motion of the center of the epicycle on the deferent orb, whose center is eccentric to the center of the World, fastness and slowness [should] occur with respect to the center of the World—as was said concerning the Sun. It therefore follows that the center of the epicycle should not cut equal arcs on the inclined [orb] in equal times. Yet such is not the case, since double elongation is not adjusted in the *zīj*es. An adjustment is not made because the center, even while moving along the circumference of the eccentric orb, cuts equal arcs on the inclined [orb] in equal time periods. If the mover of the eccentric [orb] were the inclined [orb], so that this motion would be uniform, the receding from and approaching the center of the World [by the epicycle center] would cease to exist. Therefore, one of two situations would be necessary: either [there would be] no approach of the center of the epicycle toward the center of the World nor receding from it, or else [there would be] different states in the motion of the center resulting in fastness and slowness; both of these [situations] are impermissible. This is a great doubt regarding this account that none of the practitioners of this discipline has raised any objection against, or, if anyone has, it has not reached us. There is a subtle solution for this doubt, but it is not appropriate to bring it up in this epitome. If at some other time, the blessed temper of the Prince of Iran—may God multiply his glory—would be pleased to command delving into this problem, an exposition will be made on it—God willing.

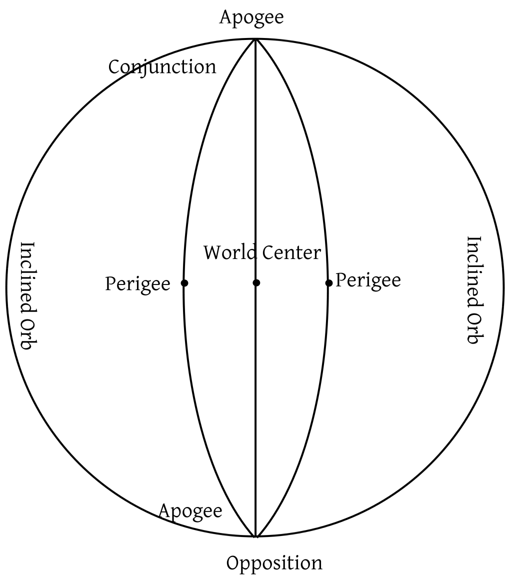
[25] This is the exposition of the configuration of the orbs of the Moon and their motions in longitude. There remains to explain the terms that are in common use among this group [of practitioners]. So, we say:

[26] **The lunar mean** is an arc on the parecliptic orb between the first of Aries and the [point of] intersection of the parecliptic with the latitude circle that passes through the endpoint of a line extending from the center of the World to the center of the epicycle and reaching the surface of the parecliptic. **The apogee of the Moon** is an arc on the parecliptic orb between the first of Aries and the [point of] intersection of the parecliptic with the latitude circle that passes through the end of a line extending from the center of the World to the apogee and reaching the surface of the parecliptic. **The center of the Moon and its double elongation** is an arc on the parecliptic orb between [the point] of intersection of the apogee's latitude circle with the parecliptic and the intersection of the epicycle center's latitude circle with the parecliptic. **The proper [anomaly] of the Moon** is an arc on the equator of the epicycle between the mean apex and the body of the Moon. **The adjusted proper [anomaly]** is an arc on the epicycle equator between the apparent apex and the body of the Moon. **The true position of the Moon** is an arc on the parecliptic orb between the first of Aries and the [point of] intersection of the parecliptic with the latitude circle that passes through the endpoint of a line extending from the center of the World to the center of the body of the Moon and reaching the surface of the parecliptic. **The node of the Moon** is an arc on the parecliptic equator between the first of Aries and the point of intersection of the inclined equator with the parecliptic equator. The illustration of the Moon's orbs is as follows:



[Figure 1]

[27] Practitioners of this science content themselves with the circles in black. From this illustration [Figure 2], the manner of the circuit of the center of epicycle in the inclined [orb,] and [the fact] that it reaches the apogee and perigee of the deferent orb twice will be understood:



[Figure 2]

[28] Other situations concerning the Moon, such as latitude, lunar eclipses, parallax, etc. will be treated later in their proper places—God willing.

CHAPTER SIX
An Exposition of the Orbs and Longitudinal Motions of the Upper Planets and Venus

[1] When the situation of the other planets is contemplated, [it will be seen that] Saturn, Jupiter, Mars, and Venus, all four, have direct and retrograde motion in common. The upper planets undergo combust at the midpoint of the direct motion period and are in opposition with the Sun at the midpoint of the retrogradation period; Venus undergoes combust in both situations. Venus's maximum distance from the Sun is not more than around 47 degrees. The direct motion of these planets is when they are distant from the Earth, and [their] retrogradation is when they are in proximity to the Earth. If one retrogradation is com-