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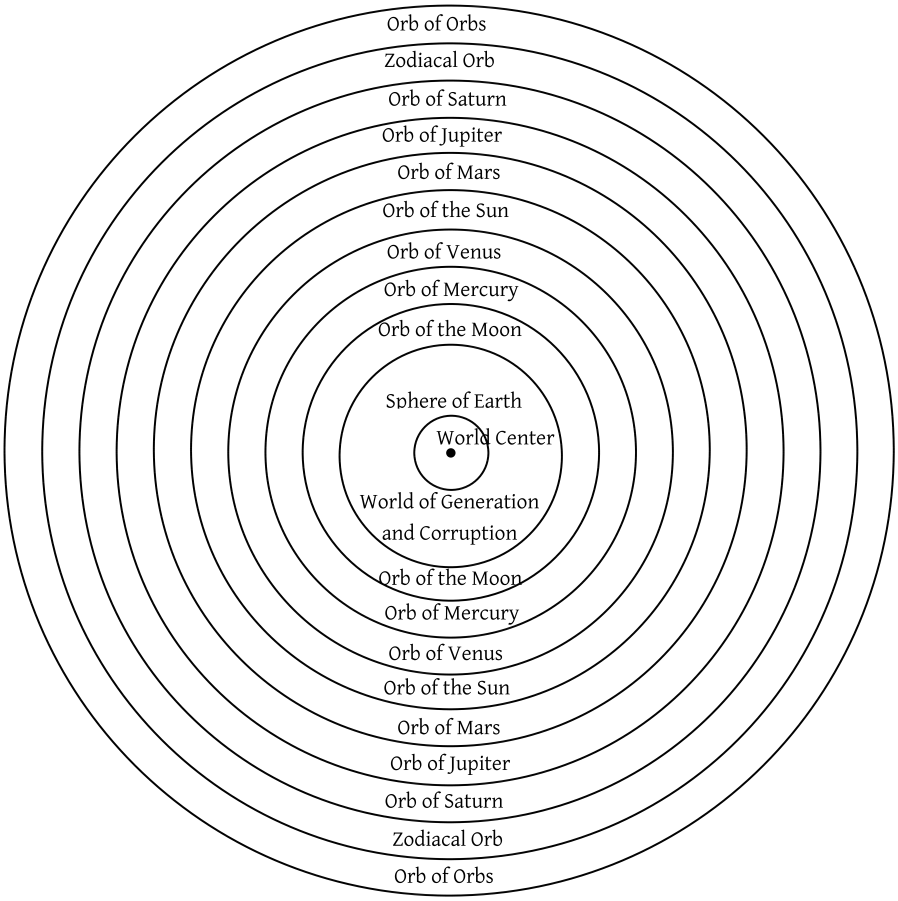

[Figure 1]

CHAPTER TWO

An Exposition of the Primary and Secondary Motions, and the Names of the Great Circles

[1] In the introduction we said that when a sphere has rotational motion, there results an equator, two poles and an axis, and that the equator is one of the great circles. Now we say that two different rotational motions in one sphere cannot be perceived, unless they are around either different poles or different centers; so that, for example: [1] the two poles of one motion are different from the two poles of the other

motion in such a way that one axis intersects the [other] axis at the center, and one equator intersects the other in two places; or [2] one motion is around one center and the other is around another center, and this necessitates that that sphere becomes two spheres, the equator of one lying in the plane of the equator of the other, the center [of one] outside the center of the other, and the [two] axes parallel. If, however, the [two] centers are one and the poles are at the self-same two points, only one motion can be perceived, and that motion amounts either to the sum of the two motions in question if both are going in the same direction, or to the difference between the faster and the slower if they are going in different directions.

[2] All celestial motions, relative to one another, are of these two types only. As for one body moving another body inside it, if the difference between the two motions is of the first type, then the poles of the internal sphere are inseparable from two specific points on the external sphere. Then, by the motion of the external sphere, those two points are displaced, and the whole [internal] sphere moves because of the displacement of its poles. If the difference is of the second type, such that the internal sphere and its center is part of the external sphere, then the former is moved by the motion of the latter.

[3] However, the moving by the orb of the sphere of aether, which is the sphere of fire, this being known from the motion of comets when, upon reaching that place, they move along with the orb—is neither of these two types. Rather, it is by way of a conformity to its own place. In other words, when [aether's] place—which is the concave surface of the orb—moves, it will move by way of conformity, since everything firmly fixed in a place will cling and adhere to it. The farther away it is [from the orb], the weaker its motion becomes, until it ceases altogether.

[4] After this introduction, we say that when the Orb of Orbs moves with the primary motion, it inevitably has an equator and two poles. Its equator is called the **equinoctial [celestial equator]**, and its two poles are called the poles of the primary motion and also the poles of the equinoctial. The reason this circle is called the equinoctial is that when the Sun reaches this circle, day and night are of equal length. Due to this motion, every point on the surface of the sphere produces a circuit parallel to the equinoctial. These circuits are called **day-circles**. These circles and circuits must be imagined on all the orbs of the stars, since they are all subject to the primary motion.

[5] When the zodiacal orb and its motion are considered, there is produced another equator that intersects the equinoctial at two places. That equator is called the **zodiacal orb**, or the **zodiacal equator**. Its two poles, which are on opposite sides of the two equinoctial poles, are called the two poles of the zodiacal equator. The planes of the equinoctial and the zodiacal equators intersect each other at acute and obtuse angles, and the maximum distance between the two equators is the maximum distance between the two poles. This is called the **maximum** or **total obliquity**.

[6] Every star has a circuit parallel to the zodiacal equator; [collectively] they are called the parallels of latitude. When a circle is imagined that passes through all four poles—i.e., the two equinoctial poles and the two poles of the zodiacal orb—it will inevitably pass through the maximum distance between the two equators. This circle is called the **solstitial colure** [lit., that passing through the four poles], and its two poles are the two points of intersection between the equinoctial and the zodiacal orb. The plane of this circle is at right angles to the planes of the first two circles. The zodiacal orb is divided into four equal parts by the equinoctial and this circle: a vernal quarter and a summer quarter—and these two quarters are north of the equinoctial equator—and an autumnal quarter and a winter quarter—and these

two quarters are to the south [of the equinoctial equator].

[7] The two points of intersection between the equinoctial and the zodiacal orb are called the **two equinox points**: the one the Sun comes to in going toward the north is called the **vernal equinox**; the other is the **autumnal equinox**. The two points of intersection between the solstitial colure and the zodiacal orb are called the **two solstice points**: the northern being summer and the southern winter.

[8] The arc on the solstitial colure falling between the two equators is called the **obliquity** and is equal to the arc between the two poles. That which is between the equator of one and the pole of the other is called the **complement of the obliquity**. If one takes a circle to have 360 parts, the total obliquity, according to the observation of Battānī and other moderns, is 23°35′, with a complement of 66°25′. In the observations made during the reign of Maʾmūn, they found 23°33′. Ptolemy found it to be 23°52′, and prior to him it was taken to be 24°0′.

[9] A group of moderns has said that since these variations are proportionally decreasing, it appears that the two equators are coming closer to each other. If this is so, then one of two things will happen: either they will continue to proceed toward each other until they reach a point at which they coincide, and day and night will be equal throughout the World, and afterwards they will pass each other, the northern half of the zodiacal equator becoming southern and the southern half northern; or else, their proceeding toward each other will have a limit and when it is reached they will move away from each other again until another limit is reached. In either case, another body must be assumed to be the principle of this motion.

[10] When one takes a [discrete] part on the orb and it is desired to find the distance of that part from the equinoctial equator, a circle should be imagined that passes through that part and also the two poles of the equinoctial equator, its plane thus being perpendicular to the equinoc-

tial equator. This circle is called the **declination circle**. What is on this circle between that part and the equinoctial equator is the distance of that part from the equinoctial equator. When this circle is assumed for the parts of the zodiacal equator, what lies on this circle between the zodiacal equator and the equinoctial equator is called the **first declination**. This circle is unique in type but infinite in individuation when one assumes one [circle] for every part, in contrast with the first three circles, which never vary anywhere in the World nor with respect to particular parts.

[11] If it is desired to relate a certain part to the zodiacal equator with respect to distance, then the circle must be considered in a way that passes through that part and through the two poles of the zodiacal equator. It is called the **circle of latitude**. What lies between that part and the zodiacal equator on this circle is called the latitude of that part, and what lies between the zodiacal and the equinoctial equators on this circle is called the **second declination**. The relation of this circle to the zodiacal equator is like the relation of the declination circle to the equinoctial equator, and the condition of this circle is like that of the declination circle, infinite in individuation but unique in type. The latitude of stars can be known from this circle. This latitude is cited with respect to longitude. For true position, longitude is needed. Longitude is the arc on the zodiacal equator between the vernal equinox point and where this [latitude] circle intersects the zodiacal equator. If a star has no latitude, the longitude is between the vernal equinox point and the center of the star. The position with respect to the zodiacal equator of a star having latitude is the point of intersection of the zodiacal equator with its latitude circle.

[12] If six circles of latitude are conceived such that the zodiacal orb is divided into twelve equal segments, like the segments of a melon, the intersection of these circles with one another being of course at the two poles of the zodiacal orb—one of these circles being the solstitial

colure and another passing through the two equinox points—then the [resulting] divisions are called the **twelve signs of the zodiac**. Each sign has a 30° longitudinal length and a 180° latitudinal width from pole to pole. On this basis, a star that is away from the zodiacal equator is said to be "in" a sign when it is in one of these divisions.

[13] These then are five of the great circles imagined on the orb that are not based upon a relation to terrestrial locations. Now when terrestrial locations are taken into consideration, the circle that separates the visible half of the orb from the invisible half is called the **horizon circle**. It has two poles: the one that is up is called the **zenith**, and the one opposite it below the Earth is called the **nadir**. Of the circles parallel to this circle that pass through loci on the orb, those that are above are called **almucantars of altitude** and those that are below are called **almucantars of depression**. If another circle is conceived passing through the two poles of the celestial equator and the two poles of the horizon circle, no doubt also at right angles both to the celestial equator and the horizon, that circle is called the **meridian circle**, and it divides the World into eastern and western hemispheres. This circle has two poles, one the east point and the other the west point. The stars reach this circle twice in every revolution: once at the midpoint of the period of visibility and once at the midpoint of the period of invisibility. They also reach the horizon circle twice: once at the time of rising and once at the time of setting. If another circle is conceived passing through the two poles of the meridian and the two poles of the horizon, no doubt also being at right angles to both the meridian and horizon, that circle is called the **east-west circle**, or circle of the **initial azimuth [prime vertical]**. It has two poles, one the north point and the other the south point.

[14] By means of these three circles, the [celestial] orb is divided into eight parts, four above and four below. One of the four is between east and north; the second between west and north; the third between west

and south; and the fourth between east and south. Each of these three circles is unique in type but numerous in individuation, according to given localities on the Earth. If one takes a celestial locus and desires to know its distance from the horizon, a circle should be taken that passes through it, namely, that locus, and the two poles of the horizon, i.e., the zenith and its opposite. This circle is called an **altitude circle**. What is between the horizon and that locus on this circle is called the [arc of] altitude. When the star reaches the meridian, this circle will coincide with the meridian; what is between this circle and the initial azimuth on the horizon circle is called the **azimuth** of the star. If the star has no azimuth, its altitude circle will be the initial azimuth. Considering the celestial loci, there are also numerous altitude circles.

[15] From this discussion, nine great circles are now known: [1] the equinoctial; [2] the zodiacal orb; [3] the solstitial colure; [4] the declination circle; [5] the latitude circle; [6] the horizon circle; [7] the meridian circle; [8] the initial azimuth circle; and [9] the altitude circle. This then is the intention of this chapter.

CHAPTER THREE
An Exposition of the Circumstances of the Eighth Orb and the Fixed Stars

[1] Every star, aside from the Sun and Moon (lit., the luminaries) and the five vacillating planets, is reckoned one of the fixed stars, and all of these stars are fixed on the eighth orb, which we call the orb of the fixed stars and the orb of the zodiacal signs. They are called "fixed" due to the slowness of their movement, i.e., fixed compared to the planets, or due to the immutability of their positions [in the sky] and with respect to one another, since these stars always maintain their latitude and move only in longitude. Their longitudinal movements were not perceived by the ancients, who reckoned them as fixed; but later, dur-