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BOOK I

On the Introductory Propositions of this Science Which Comprises Two Chapters

CHAPTER ONE

On the Introductory Propositions Pertaining to the Science of Geometry

[1] Anything that can be perceptibly indicated and is not divisible is called a **point**. What can be divided in one direction, such as in length only but not in width or depth, is called a **line**. What is divisible in two directions, such as in length and width but not in depth, is called a **surface**. What is divisible in all three directions is called a **solid**.

[2] A **straight line** is one on which all given points are facing one another. A **circular line** is one that has a uniform curvature, such as the circumference of a circle. Similarly, a **plane surface** is that [surface for which] the lines that are assumed on it are all straight, whether in length or in width.[1] A **circular surface** is one that has a uniform curvature, such as the surface of a sphere.

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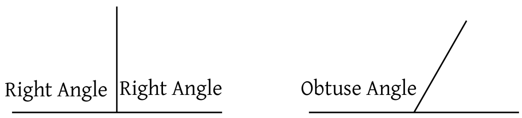
1. MSS F, G (Revised version): "Similarly a plane surface is one on which straight lines may be assumed in all directions." Cf. *Tadhkira*, I.1[2], 1:92-93 (Baghdad version).

[3] The end and the beginning of a line may be at a point. A finite straight line necessarily has a beginning and an end, whereas sometimes it is possible for a circular line to have no beginning or end, as in the case of the circumference of a circle. A surface may end at lines, and a finite plane surface necessarily has edges, but a circular one may sometimes not, such as the surface of a sphere. A solid necessarily ends at a surface.

[4] When one straight line meets another straight line and they are not aligned, from their intersection two angles occur, i.e., two surfaces coming together at one point. If the two angles are equal, each is called a right angle. If they are unequal, the smaller is called acute, and the larger obtuse, as in this illustration:

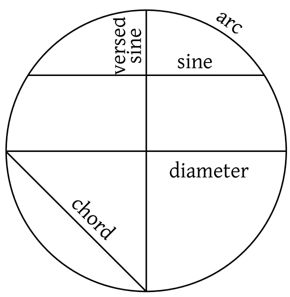


[Figure 1]

[5] When two lines in the same plane do not meet and if extended without end in both directions do not intersect one another, they are called parallel. For surfaces, the right, acute and obtuse angles, and parallelism, are analogous.

[6] A circle is a surface bounded by one circular line in such a way that at the middle of that surface a point can be assumed from which all straight lines drawn to the [circular] line are equal. The point is called the center of the circle, and the line is called the circumference. Any part of the circumference is called an **arc**. A straight line joining the two endpoints of an arc is called a **chord**. A line extending at right angles from the midpoint of the chord to the circumference is called a **versed sine**. A surface that is cut off from a circle by an arc and a straight line is called a **circular segment**. The **diameter** of a circle is a line that cuts a circle into two [equal] halves and necessarily passes

through the center; it is the greatest chord. A **sine** [of an arc] is half the chord of twice the arc. Here is an illustration of a circle and the lines [that pertain to it]:



[Figure 2]

[7] A **perpendicular** is a line rising from a line or plane such that the angles produced are right.

[8] A **sphere** is a solid bounded by one circular surface within which a point can be conceived such that all lines extending from that point to the surface are equal. That point is **the center of the sphere**, and those lines are **radii.** If it is assumed that the sphere rotates, two points on the surface of the sphere, on two sides, do not move around as the sphere rotates; those two points are called the two **poles.** The diameter between these two points, which also does not move, is **the axis of the sphere.** Any point assumed on the surface of the sphere will produce a circle when a rotation is completed and that point returns to its [initial] position. That circle is called **the circuit of that point.** The plane of any one of those circles will divide the sphere into two parts, one larger and the other smaller, except for the one circuit that is midway between the two poles and divides the sphere into two equal halves: it is called **the equator of the sphere.** Any circle assumed on the surface of the sphere that divides the sphere into two [equal] halves is called **a great circle.** The two points that serve as the two poles of such a circle are called **the poles of that circle.** The center for each and every circuit is on the axis, and circuits are parallel to each other. Two circuits

are equal in size when the distance of one of them from a pole is equal to the distance of the other circuit from the other pole. When any great circle passes through the two poles of another great circle, their surfaces intersect one another at right angles. [Great circles] that do not pass through each other's two poles intersect at acute and obtuse angles. In any case, any two great circles assumed on a sphere intersect one another at two points. These two points are called the two points of intersection. The maximum distance between those two circles should be equal to the maximum distance between the two poles.

[9] An **orb** is a body bounded by two circular surfaces, one inside and the other outside, the center of both surfaces being the same point. [The orb (*falak*)] has been likened to the whorl (*falaka*) of a spindle. Of the two surfaces, one is called the convex and the other the concave. By borrowing [the term], circles are also called orbs.

[10] A **circular cylinder** is a body whose base and top are two equal and parallel circles and which is bounded by a circular surface. A line serving as the axis stands at right angles to the two circles, and that line is called its **sagitta** (*sahm*).

[11] A **circular cone** is a body whose base is a circle and whose top is a point. The line from that point to the center, i.e., the sagitta of the cone, is perpendicular to the plane of the circle. It is also called a **pine-shaped cone**. This is all that is necessary to present in this chapter.

CHAPTER TWO
On the Introductory Propositions Pertaining to the Science of Natural Philosophy

[1] The introductory propositions that have been demonstrated in the science of natural philosophy, and that are used in this science by way of [accepted] principles without being demonstrated, are as follows: