Naṣīr al-Dīn Muḥammad al-Ṭūsī

al-Risāla al-Muʿīniyya
(al-Risāla al-Mughniya)
and
its Supplement

Volume II

English Translation

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Miras-e Maktoob
Tehran, 2024

Miras-e Maktoob
346

Science, Art & Technology
25

Texts and Studies in the History of Science
(2)
Series Editor
Hossein Masoumi Hamedani

Written Heritage Research Institute, 2024
First Published in I. R. of Iran by Miras-e Maktoob

ISBN 978-600-203-204-1
ISBN (Vol.2) 978-600-203-288-1

PRINTED IN TEHRAN

In the Name of God, the Compassionate, the Merciful

Oceans of Iranian and Islamic culture lie in manuscript form. These manuscripts are not only the record of the achievements of our nation's great scholars, they are also testimonials to our unique national identity. It is, therefore, the duty of every generation of Iranians to protect and celebrate this priceless heritage and to spare no effort in restoring these records on which all studies of Iran's history and culture depend.

Many efforts towards better identification, study, and preservation of our country's manuscript collections have been launched. In spite of these efforts, and despite the fact that hundreds of books and treatises that deal with this important area of learning have been published, much remains undone. Thousands of books and treatises either linger as unidentified codices in Iranian and foreign libraries, or await publication. Others, although previously published, exist in unsatisfactory editions and need to be re-edited according to modern scholarly standards.

It is the duty of scholars and cultural organizations to undertake the important tasks of restoring and publishing these manuscripts. The Written Heritage Publication Center was established in 1993 in order to achieve this important cultural objective with the purpose of supporting the efforts of scholars, editors, and publishers who work in this field of learning. We hope that by suppoting scholarly work in this area, we can help make an essential collection of scholarly texts and sources available to the scholarly community that is engaged in the study of Iran's Islamic culture and civilization.

The Written Heritage Research Institute (Miras-e Maktoob)

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Ḥall-i Mushkilāt-i Muʿīniyya

Preface

It is with great pleasure that we (Fateme Savadi, Sajjad Nikfahm-Khubravan and myself) present this translation of Naṣīr al-Dīn al-Ṭūsī's *al-Risāla al-Muʿīniyya* and its "Supplement" (or "Appendix"), the *Ḥall-i Mushkilāt-i Muʿīniyya,* based on the critical edition prepared by Sajjad Nikfahm-Khubravan and Fateme Savadi.[1] It would be a considerable understatement to say that it has been long in the making.

I began the translation in collaboration with Prof. Wheeler Thackston of Harvard University in the early 1980s. At the time, we worked mainly on the *Ḥall* and produced a first draft; then, much to my pleasant surprise, Thackston sent me a translation in 1992 that included the entire *Muʿīniyya* and a revision of our *Ḥall* translation. Unfortunately,

1. Naṣīr al-Dīn Muḥammad al-Ṭūsī, *al-Risāla al-Muʿīniyya (al-Risāla al-Mughniya) and its Supplement*, Volume I: Critical Edition of the Persian Texts, edited by Sajjad Nikfahm-Khubravan and Fateme Savadi (Tehran: Written Heritage Research Institute [Miras-e Maktoob], 2020). Also available as open access at `https://ismi.mpiwg-berlin.mpg.de/page/muiniyya-edition-2020` and `https://escholarship.mcgill.ca/concern/books/qr46r555w?locale=en`.

other obligations kept intervening, and many years went by without completing the project. But as it turned out, Fateme Savadi and Sajjad Nikfahm-Khubravan, both McGill graduate students at the time, were able to produce their critical edition. Although it became clear that considerable emendations needed to be made to Thackston's translation, nevertheless his draft helped us considerably as we finalized the translation. We wish to sincerely thank him for his generosity in allowing us to benefit from that translation and acknowledge his role in what appears here.

Before discussing the present translation, it is important that we reiterate what is meant by a critical edition of the *Muʿīniyya* and its Supplement. (For a fuller discussion, please see the introduction to the edition.) Ṭūsī completed the first version of the *Muʿīniyya* in 632 H/1235 CE. The *Ḥall* was added as a supplement[1] in 643 H/1245 CE. But this isn't the end of the story. Ṭūsī continued making emendations, the most dramatic being revisions of the introductions to both works in which he removed any mention of his original patrons, the Ismāʿīlī rulers of Kūhistān in eastern Iran. This, of course, came after Iran fell under the rule of the Mongols and Ṭūsī's subsequent employ by them. Through rather impressive codicological analysis, Nikfahm-Khubravan and Savadi were able to determine what we call the oldest Ismāʿīlī version of the text, which is the one presented in their edition. This is as close as we can come at present to Ṭūsī's original version. Their critical apparatus allows the reader to ascertain the later versions of the texts.

1. Some manuscripts refer to it as an appendix (*dhayl*).

The choice of the original version for the edition, rather than, say, the revised version that expunged the Ismāʿīlī references, was made for a number of reasons. First, it allows us to ascertain Ṭūsī's earliest formulations about astronomy and the discoveries of his new models that employ the "Ṭūsī-couple." Second, it provides invaluable information about Ṭūsī's patronage at the Ismāʿīlī court and how that might have influenced how he presented his material. Third, although the revised versions are mostly the same as the originals (with the exception of the introductions), there still are a number of revisions that are part of the textual history, some due to Ṭūsī, some perhaps to others. What we can deduce from some of these revisions is that Ṭūsī made a number of "mistakes" in his original draft; by mistakes, we mean obvious orthographic, numeric, grammatical, and conceptual errors that were changed in later versions, again either by Ṭūsī, students, scribes or someone else. Since our best witness to the Ismāʿīlī version, University of Tehran MS 1346, also contains Ṭūsī's later revisions, we have concluded that Naṣīr al-Dīn, due to haste or otherwise, was not as meticulous as he would later be when he composed the Arabic version of the *Muʿīniyya*, i.e., *al-Tadhkira fī ʿilm al-hayʾa* (*Memoir on Astronomy*).

Now this creates a dilemma for a translator when confronted with obvious mistakes, such as 332,460,952 instead of the correct 33,460,952 (IV,5[7]). The temptation is to assume this to be a copyist error and put the correct number in the translation. But because it is attested in the witnesses from which the Ismāʿīlī version has been determined, it was decided to keep the incorrect number in both the edition and translation, with corrections in the notes; as best as we can now determine, this number (and similar cases) is due to Ṭūsī. Likewise with several numbers in the listings of constellations: our best witnesses attest to the "mistakes," although other witnesses have attempted corrections. To complicate matters further, Ṭūsī himself in his *Taḥrīr al-Majisṭi* (recension of the *Almagest*), completed shortly after the *Ḥall* in 644 H/1247

CE, has the correct numbers for the constellations, i.e., those that correspond to what one finds in the *Almagest*. Again, our decision was to translate what is written in our best witnesses. In that way, the translation follows the critical edition.

A similar problem pertains to the figures. Again, the temptation is to "correct" what is found in the manuscript witnesses, but this potentially leads to considerable distortions and misrepresentations, as has been pointed out with respect to Johan Ludvig Heiberg's editions of Greek mathematical texts.[1] Generally the figures have been drawn following what is found in University of Tehran MS 1346, but we recognize that they contain errors that may or may not be due to Ṭūsī. (An example is Figure 5 in Section 7 of the *Ḥall*.) In order to assist the reader in comparing our figures with those in manuscript witnesses, we plan to put images of the figures online with the edition and translation, once we have obtained permissions from the various repositories.

The translation follows the standard procedures that we have used for Ṭūsī's *Tadhkira*,[2] Quṭb al-Dīn al-Shīrāzī's *Nihāyat al-idrak*,[3] etc. We have kept notes to a minimum; and a third volume is anticipated that will provide a study and commentary.

Finally, I wish to acknowledge the support of the US National Sci-

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1. See Ken Saito and Nathan Sidoli, "Diagrams and Arguments in Ancient Greek Mathematics: Lessons Drawn from Comparisons of the Manuscript Diagrams with Those in Modern Critical Editions," 135-62, and Reviel Netz, "The Texture of Archimedes' Writings: Through Heiberg's Veil," 163-205, in *The History of Mathematical Proof in Ancient Traditions*, ed. Karine Chemla (Cambridge: Cambridge University Press, 2012).

2. F. J. Ragep, *Naṣīr al-Dīn al-Ṭūsī's Memoir on Astronomy (al-Tadhkira fī ʿilm al-hayʾa)*, 2 vols. (New York: Springer-Verlag, 1993).

3. Fateme Savadi, "The Historical and Cosmographical Context of *Hayʾat al-arḍ* with a focus on Quṭb al-Dīn Shīrāzī's *Nihāyat al-Idrāk*" (PhD diss., McGill University, 2019).

ence Foundation (Grant no. SES9911005) for the years 2000-2001 that allowed me to work on the translation. And we thank Sally P. Ragep, whose editorial assistance and interventions helped make this a much better translation than it otherwise would have been.

Jamil Ragep
April 2022
Chicago, Illinois USA

al-Risāla al-Muʿīniyya
(*al-Risāla al-Mughniya*)

Ismāʿīlī Preface

In the Name of God, the Compassionate, the Merciful

[1] **Gratitude and praise be to the Mighty Presence of the Possessor of Majesty, the lights of the subtleties of whose Wisdom shine from every single particle of the universe, and the effects of the marvels of whose Power radiate from every single part of existent things; to the Powerful who brought forth from the forming mold with the hand of destiny so many luminous bodies; to the Subduer who caused the many spherical bodies to wander submissively in the revolving whirlpool; to the Ordainer who turned one lower point into the center of the equator of the upper circuits; to the Regulator who made a handful of earth the basis for the positions of the stars and orbs to**[1] bring into order the rules of both existences and to complete the deficiencies of both worlds. [He who] also illuminated the up and down of the irrealis world by means of a body that stands as the pupil of all creation, and embellished both the beginning and end of the real existence by an individual who is the best of the rank of men of insight, so that the ways and places of ascent towards perfection and consummation, and superiority and generosity, in the stages and ranks of the universe of multiplicity and unity, of which the world of the unseen and the seen

1. Words in bold represent text that is common between the Ismāʿīlī Preface and the Revised Preface.

is said, became ordered and determined, **and the proofs of the oneness of God and means of [His] abstraction, which includes the ways to supplicate to the Originator and Restorer and the pathways of attaining [knowledge of] the provenance and destination, were made clear and demonstrated. "Is not His the creation and the command? Blessed be God, the Lord of the worlds."**[1]

[2] His majesty's court, that mine of absolute light and source of manifestation of the Truth, that truly is nothing other than the sacred threshold and exalted throne of the lord of the world, commander of the sons of Adam, manifestation of divine decree, source of divine mercy, center of the circle of existence, the embodiment of the knowledge of the Necessary of Existence, lord of the age and interpretation of the Merciful, ʿAlāʾ al-Dunyā wa-al-Dīn, glory of Islam and Muslims, shadow of God in the Worlds and His proof to all creation—may God elevate his decree and bless his remembrance—may that court be enveloped by all manner of prayers and all types of greetings, and may the reins of the vicissitudes of fortunes be directed away from the *Kaʿba* of the followers of unity and away from the Qibla of the lords of knowledge, by the True One and Possessor [of the Truth].

[3] The purpose of this opening supplication and ordered invocation is that ever since the writer of this draft attained the honor of proximity to the triumphal arch of the sublime court of the king, the knowledgeable, the just, the most perfect, the most erudite, the most equitable, the most triumphant, the most victorious, the most diligent, the most noble, Nāṣir al-Ḥaqq wa-al-Dīn, the felicity of Islam and Muslims, the most just of kings and sultans, spreader of beneficence throughout the worlds, king of kings of the Arabs and Persians, most excellent wielder of the sword and the pen, sultan of the chiefs of the orient and the occident, powerful [one] of the holy threshold (*ʿAzīz al-Ḥaḍra al-Muqaddasa*)

1. Cf. Quran, 7:54.

[of the Ismāʿīlī Imām], *Chosroes* of the horizons, commander of the world, foundation of the universe, King of Iran, ʿAbd al-Raḥīm ibn Abī Manṣūr—may God cause his kingdom to endure forever and magnify his greatness—and has been honored and blessed by the privilege of benefiting and the dignity of seeking grace from that court wherein is a haven for Saturn, his resolution was devoted to attain the fortune of serving the Prince of Iran, pride of the world and worldlings, most magnificent chief, exemplar of the chiefs of the Arabs and Persians, Muʿīn al-Dawla wa-al-Dīn,[1] protector of Islam and Muslims, crown of kings and sultans, treasure of grandees and erudite men in the worlds, personification of nobility and honor, revivifier of the forebearers' customs of excellency, the distillation of noble virtues, the best of the nobles of the horizons, the most noble descent in the world, pride of the world, the most generous and the noblest of Iran, Abū al-Shams son of ʿAbd al-Raḥīm—may God magnify his glory and perpetuate his ascent to the apogee of glory and nobility—the fame of whose greatness and excellence has spread throughout all horizons and regions, and the truth of the dictum, "He who resembles his father cannot be unjust," is apparent upon his brilliant forehead, and his [Ṭūsī's] devotion was dedicated to waiting for facilitating a means that ensures the perception of that nobility. With the extension of the period of deprivation and the unfavorable requirements of the time, the mind was preoccupied with the thought that through what means the manifestation of sincerity to his court might be inaugurated, or through what expedient something of my goodwill and sincerity might be offered, and the aim of my will and the extent of the ambition comprised of this desire, which is, in the desirer's eye, the noblest of all.

1. Muʿīn al-Dawla wa-al-Dīn (lit., helper of the state and religion) is the name of Nāṣir al-Dīn Muḥtasham's son, whence the title of the treatise, *Muʿīniyya*.

[4] It was in the midst of such contemplation and in the wilderness of such perplexity that the dawn of the desire broke forth and from that house of glory and nobility and that family of excellent dispositions an order was issued to this sincere and supportive servant to compose several chapters explaining the circumstances of the bodies [of the World] and something of the knowledge of the stars, [i.e., astronomy] and the judgments [of the stars, i.e., astrology]. Although this hapless, incapable one was aware that his own shortcomings in ability and lack of comprehension of the discipline, as well as other causes for deficiencies such as distractions and preoccupations, were numerous, nonetheless since in conforming to the command the long-lasting wish and long-standing desire was being embodied in undertaking this service and in fulfilling the requirements of servitude, the initiation of this task became inevitable.

[5] Therefore, it seemed appropriate to put together hastily a work containing a summary of the science of configuration (*ʿilm-i hayʾat*) and to send it to his presence as a gift and thereafter to engage with the other disciplines [of astronomy] unhurriedly. In accordance therewith the writing of this compendium was begun, and it was named *al-Risāla al-Muʿīniyya* (The Muʿīniyya treatise). It is hoped that the Lord of Glory will grant success and guard and protect in all endeavors. It is expected from the benevolence of his mighty majesty that if errors or slips of the pen are observed, due to the apology that precedes, and after being so kind as to correct them, to grant forgiveness, which is one of the customs of greatness, and to couple this audacity with overlooking. May God—may He be glorified and exalted—increase his fortune and exaltedness and make his status and magnificence ever increasing—He is the answerer of prayers.

Revised Preface

In the Name of God, the Compassionate, the Merciful

[1] Gratitude and praise be to the Mighty Presence of the Possessor of Majesty, the lights of the subtleties of whose Wisdom shine from every single particle of the universe, and the effects of the marvels of whose Power radiate from every single part of existent things; to the Powerful who brought forth from the forming mold with the hand of destiny so many luminous bodies; to the Subduer who caused the many spherical bodies to wander submissively in the revolving whirlpool; to the Ordainer who turned one lower point into the center of the equator of the upper circuits; to the Regulator who made a handful of earth the basis for the positions of the stars and orbs to regulate the rules of both existences and to bring in order the ranks of both worlds. Just as He illuminated the encompassing surface of the irrealis world with the luminous heavenly bodies, he adorned the center of the upper bodies with the true lights of guidance and teaching of the prophets and holy men so that the proofs of the oneness of God and means of [His] abstraction, which includes the ways to supplicate to the Originator and Restorer and the pathways of attaining [knowledge of] the provenance and destination, were made clear and demonstrated.[1] "Is not His the

1. In addition to the "Revised Preface," some manuscripts have a shorter preface consisting of only the first paragraph of the "Revised Preface."

creation and the command? Blessed be God, the Lord of the worlds."[1]

[2] And gifts of prayers and presents of salutations of the Divine Majesty be upon the seal of the prophets, the essence of the purest, Muḥammad the chosen one, and to his family, companions, progeny, and loved ones, eternally and perpetually.

[3] After setting forth this opening, one should know that this book is a concise treatise on the science of the configuration (*hayʾat*) of the upper and lower bodies and the knowledge of distances and sizes in an abridged way. It was written in haste upon the request of some friends, and it was named *al-Risāla al-Mughniya* (The sufficing treatise). If the great ones will ennoble it through the honor of their study, convey the correction of any mistakes they find, this will not be unbecoming of their generosity. May the glorified God, by His grace and kindness, grant success and support, and protection and strength in all circumstances!

1. Cf. Quran, 7:54.

Index of Chapters of This Treatise

Index of Chapters of this Treatise

It is appropriate to arrange this treatise into four books, and each book consists of several chapters, as follows.

This is the Index of Chapters.

The Text of *al-Risāla al-Muʿīniyya*

BOOK I

On the Introductory Propositions of this Science Which Comprises Two Chapters

CHAPTER ONE

On the Introductory Propositions Pertaining to the Science of Geometry

[1] Anything that can be perceptibly indicated and is not divisible is called a **point**. What can be divided in one direction, such as in length only but not in width or depth, is called a **line**. What is divisible in two directions, such as in length and width but not in depth, is called a **surface**. What is divisible in all three directions is called a **solid**.

[2] A **straight line** is one on which all given points are facing one another. A **circular line** is one that has a uniform curvature, such as the circumference of a circle. Similarly, a **plane surface** is that [surface for which] the lines that are assumed on it are all straight, whether in length or in width.[1] A **circular surface** is one that has a uniform curvature, such as the surface of a sphere.

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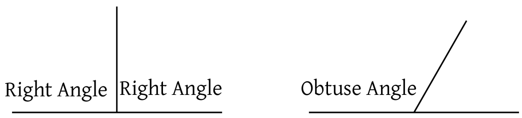
1. MSS F, G (Revised version): "Similarly a plane surface is one on which straight lines may be assumed in all directions." Cf. *Tadhkira*, I.1[2], 1:92-93 (Baghdad version).

[3] The end and the beginning of a line may be at a point. A finite straight line necessarily has a beginning and an end, whereas sometimes it is possible for a circular line to have no beginning or end, as in the case of the circumference of a circle. A surface may end at lines, and a finite plane surface necessarily has edges, but a circular one may sometimes not, such as the surface of a sphere. A solid necessarily ends at a surface.

[4] When one straight line meets another straight line and they are not aligned, from their intersection two angles occur, i.e., two surfaces coming together at one point. If the two angles are equal, each is called a right angle. If they are unequal, the smaller is called acute, and the larger obtuse, as in this illustration:

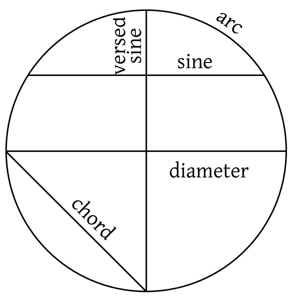


[Figure 1]

[5] When two lines in the same plane do not meet and if extended without end in both directions do not intersect one another, they are called parallel. For surfaces, the right, acute and obtuse angles, and parallelism, are analogous.

[6] A circle is a surface bounded by one circular line in such a way that at the middle of that surface a point can be assumed from which all straight lines drawn to the [circular] line are equal. The point is called the center of the circle, and the line is called the circumference. Any part of the circumference is called an **arc**. A straight line joining the two endpoints of an arc is called a **chord**. A line extending at right angles from the midpoint of the chord to the circumference is called a **versed sine**. A surface that is cut off from a circle by an arc and a straight line is called a **circular segment**. The **diameter** of a circle is a line that cuts a circle into two [equal] halves and necessarily passes

through the center; it is the greatest chord. A **sine** [of an arc] is half the chord of twice the arc. Here is an illustration of a circle and the lines [that pertain to it]:



[Figure 2]

[7] A **perpendicular** is a line rising from a line or plane such that the angles produced are right.

[8] A **sphere** is a solid bounded by one circular surface within which a point can be conceived such that all lines extending from that point to the surface are equal. That point is **the center of the sphere**, and those lines are **radii**. If it is assumed that the sphere rotates, two points on the surface of the sphere, on two sides, do not move around as the sphere rotates; those two points are called the two **poles**. The diameter between these two points, which also does not move, is **the axis of the sphere**. Any point assumed on the surface of the sphere will produce a circle when a rotation is completed and that point returns to its [initial] position. That circle is called **the circuit of that point**. The plane of any one of those circles will divide the sphere into two parts, one larger and the other smaller, except for the one circuit that is midway between the two poles and divides the sphere into two equal halves: it is called **the equator of the sphere**. Any circle assumed on the surface of the sphere that divides the sphere into two [equal] halves is called **a great circle**. The two points that serve as the two poles of such a circle are called **the poles of that circle**. The center for each and every circuit is on the axis, and circuits are parallel to each other. Two circuits

are equal in size when the distance of one of them from a pole is equal to the distance of the other circuit from the other pole. When any great circle passes through the two poles of another great circle, their surfaces intersect one another at right angles. [Great circles] that do not pass through each other's two poles intersect at acute and obtuse angles. In any case, any two great circles assumed on a sphere intersect one another at two points. These two points are called the two points of intersection. The maximum distance between those two circles should be equal to the maximum distance between the two poles.

[9] An **orb** is a body bounded by two circular surfaces, one inside and the other outside, the center of both surfaces being the same point. [The orb (*falak*)] has been likened to the whorl (*falaka*) of a spindle. Of the two surfaces, one is called the convex and the other the concave. By borrowing [the term], circles are also called orbs.

[10] A **circular cylinder** is a body whose base and top are two equal and parallel circles and which is bounded by a circular surface. A line serving as the axis stands at right angles to the two circles, and that line is called its **sagitta** (*sahm*).

[11] A **circular cone** is a body whose base is a circle and whose top is a point. The line from that point to the center, i.e., the sagitta of the cone, is perpendicular to the plane of the circle. It is also called a **pine-shaped cone**. This is all that is necessary to present in this chapter.

CHAPTER TWO
On the Introductory Propositions Pertaining to the Science of Natural Philosophy

[1] The introductory propositions that have been demonstrated in the science of natural philosophy, and that are used in this science by way of [accepted] principles without being demonstrated, are as follows:

[2] (a) A body is either simple or composite. A simple is that which is not made up of bodies of different natures or forms. A composite is the opposite of this. Necessarily composites are composed of simples. Simples are of two types: celestial and elemental. The celestials are all the orbs and stars. The elementals are those fourfold substances that are the bases of the world of generation and corruption, i.e., fire, air, water, and earth. The composites are of four types: (1) that whose composition is not complete, such as clouds, wind, shooting stars, and the like. These are called upper phenomena; (2) that whose composition is complete, i.e., it can remain for a period of time and have the capacity to retain its form, but it is not subject to growth. This is called mineral; (3) that whose composition is complete but nonetheless has the capacity for growth. This is called vegetal; (4) that which has in addition to the capacity for growth, the capacity for perception and voluntary movement. This is called animal. The latter three types are called the three engendered [kingdoms]: the fourfold elements are the mothers of these engendered, and the celestial bodies are the fathers. The elements and composites are called lower bodies, and the orbs and stars are called the upper bodies.

[3] (b) Motions are also of two types: one simple, the other composite. Simple motions are of three types: (1) that from the circumference toward the center, and what has this motion is called heavy; (2) that from the center toward the circumference, and what has this motion is called light. Each of these two motions is rectilinear; and (3) that around the center. This motion is circular. In the science of natural philosophy, it has been demonstrated that circular motions are by essence prior to rectilinear motions, i.e., if there is no circular motion there will be no rectilinear motion. The upper bodies can have circular motion but cannot have rectilinear motion, for which reason the philosophers say that the orbs and stars are neither light nor heavy and that rectilinear motion is proper to what is in the world of gen-

eration and corruption. Two of the elements, fire and air, are light; and two, water and earth, are heavy. The heaviness and lightness of composites are according to [their] composition, i.e., that in which the heavy parts are greater is heavy, and that in which the light parts are greater is light.

[4] (c) Every motion must have a principle, which is called the mover of the moving body. If the mover of a body is not outside of the body itself, the motion is attributed to that body. If it is outside, the body is said to be moved by another body. It is not possible for one simple body to be the principle of two different motions. Therefore, for every motion a mover must be established. Those bodies that are set in motion externally terminate at those bodies that are not set in motion externally.

[5] (d) No simple body in which there is a principle of circular motion, such as the celestials, may receive rectilinear motion. This being so, it is not allowable for the celestials to tear and mend. Therefore, the motion of the stars in the orbs cannot be like the motion of a fish in water; rather, for every star an orb must be established by whose motion it is moved. If a star has a particular motion of its own, [this motion] will also be circular about itself.

[6] (e) In circular motions, stopping, turning, reversing direction, and intensifying and its opposite are not permissible, but rather they are always monoformly continuous in the direction toward which the inclination (*mayl*) is directed. These are the introductory propositions that are the principles of this science, the verification of which pertains to the science of natural philosophy and metaphysics—God is all-knowing.

BOOK II
On the Configuration of the Upper Bodies, in Fourteen Chapters

CHAPTER ONE
On the Situation of All the Simple Bodies

[1] Natural Philosophers have proven (*ḥujjat*) that simple bodies can only have a spherical shape, since other shapes require a difference of parts. However, the practitioners of this science establish the circularity of bodies by sense perception, by observational testing (*iʿtibār-i raṣd*), and by proofs (*dalīl-hāyī*) that are based upon observations. Now, when they looked at the stars and the two luminaries, they found them all to be in motion from east to west such that in every nychthemeron they make one complete rotation. They found an apparent point, called the pole, around which the stars move. That which is near [the pole] does not rise or set; the farther the distance from it, the larger the circuit becomes, and all the circuits are parallel until a star is reached that rises and sets. That which is farther from the pole, the period of its invisibility becomes longer proportionally, until a star is reached whose period of invisibility is equal to its period of visibility. That which becomes farther from it in the other direction, its invisibility

becomes greater than its visibility until a star is reached whose visibility in a nychthemeron is not more than momentary.

[2] Furthermore, stars ascend gradually until the line of the meridian is reached, and from there gradually descend until they disappear. A star's size under all circumstances appears equal. It does not at times become smaller and at [other] times larger, since change in size indicates a variation in distance, with the exception of when it is quite close to the horizon, at which time it appears larger because of the density of vapors—just as a grape looks larger in water—because [apparent] size changes due to clarity or turbidity of the air.

[3] In view of these considerations, it became known that the sky is spherical, since these shapes and situations can arise only in a sphere. Upon reflection, they found the Earth as well to be spherical, because as one traverses the distance toward the north the altitude of the stars that are of permanent visibility increases and some of those that had risen and set become permanently visible; toward the south, some of the stars become permanently invisible. If one goes the other way toward the south, the stars that had been permanently invisible begin to be visible and the stars that had been permanently visible begin to have [a period of] invisibility.

[4] If one moves toward the east or west, rising and setting times become earlier or later, since the stars in eastern lands rise earlier than they do in western lands. The truth of this becomes known by the observation of celestial phenomena such as solar and lunar eclipses and shooting stars; for if two persons in two different cities make observations, the one in the city to the east will see things at earlier times than the one in the city to the west.

[5] Therefore, by this and similar evidence, it became known that the Earth is also spherical and that the heavens bounds the Earth on all sides, since if the heavens did not bound it, it would not be possible for

stars to disappear and appear and to return to their points of origin.

[6] Then too, by similar considerations, it became known that the Earth is in the middle of the heavens in a central position, because if it were closer to the upper part of the heavens, the period of visibility of the stars which are on the equator of [the first] motion would be less than their invisibility; and if closer to the lower part, the period of invisibility would be less. But this is not so, since every star that lies at the midpoint between the two poles has equal visibility and invisibility, while those that are closer to the north have a longer visibility and those that are closer to the south have a longer invisibility. When two stars are equidistant in the two directions, the invisibility of one is equal to the visibility of the other. If the Earth were closer to the north or to the south, then when night and day are equal and the Sun is at the midpoint between the two poles, the rising and setting points would not be directly opposite each other and shadows at sunrise and sunset would not coincide on one straight line. [Furthermore,] solar and lunar eclipses would not necessarily be the way they are, as will be explained. If the Earth were closer to the east or west, the length of the first half of the day would not be equal to the second half of the day; indeed, the Sun and stars would stay longer in one half of their periods of visibility and the stars would not appear equal in both directions but would appear smaller at one end. It therefore became known from this evidence that the Earth is placed in the middle of the heavens and does not incline in any direction.

[7] When, with respect to this state of affairs, observational instruments were constructed and testing was also done with them, it became certain that these circumstances are as described, and no doubt remained. Since the Earth is posited as the center of the World, and heavy bodies are inclined toward the center and light bodies are inclined toward the circumference, therefore: wherever they are, animals have their heads toward the heavens and their feet toward the

Earth; and the direction of the heavens is taken to be "up," and the direction of the Earth is taken to be "down" such that if we imagine a hole through the Earth and two people standing at either end of the hole, each would think the other was beneath himself. Hence it became known that the distances between the tops of elevated objects is greater than the distances between their bottoms, since the bottoms are closer to the center; therefore every object can be taken to be the endpoint of an Earth diameter. Furthermore, the apparent surface of water and liquids that remain still on the surface of the Earth is convex, like the surface of a sphere. The import of this is evident to sailors, who first see the tops of elevated bodies and later see their bottoms. If the surface of the water were level, they would see them all at once. Since the surface of a sphere is such that the less its distance from the center, the greater the convexity of the surface, it follows necessarily that, for example, if a vessel on top of a mountain is filled with water and the same vessel is filled with water at the bottom of a well, there will be more water at the bottom of the well than at the top of the mountain. This is one of the questions put to this group [of practitioners] by way of examination.

[8] Mountains, elevations, and depressions do not keep the Earth from being circular, because these things have no measure in relation to the Earth. One scholar has said: "I made an investigation by way of measurements, [finding] the largest known mountain to be in relation to the Earth in the proportion of one-fortieth of a millet grain to a sphere that is a cubit (*gaz*) in diameter."

[9] Turning to the order of the bodies: When the stars were investigated, nine types of motion were found. At first glance, there is one motion that can be seen right away, and it is the diurnal motion that causes the rising and setting of the stars. It is called the **primary motion**, and all the stars share in this motion. The second motion, which became known with much investigation and testing—which some of

the ancients did not recognize—is a motion that takes 36,000 years to complete one revolution according to one group of the ancients. According to modern observations, it takes place in 24,000 years, and it is called the **motion of the fixed stars**. Although other planets share this motion with the fixed stars, nonetheless it has been attributed to the fixed stars because they are distinguished by this motion. The other seven motions are the motions of the seven wandering planets, each of which has a motion in addition to these two. For this reason, nine bodies have been posited, i.e., nine orbs encompassing each other.

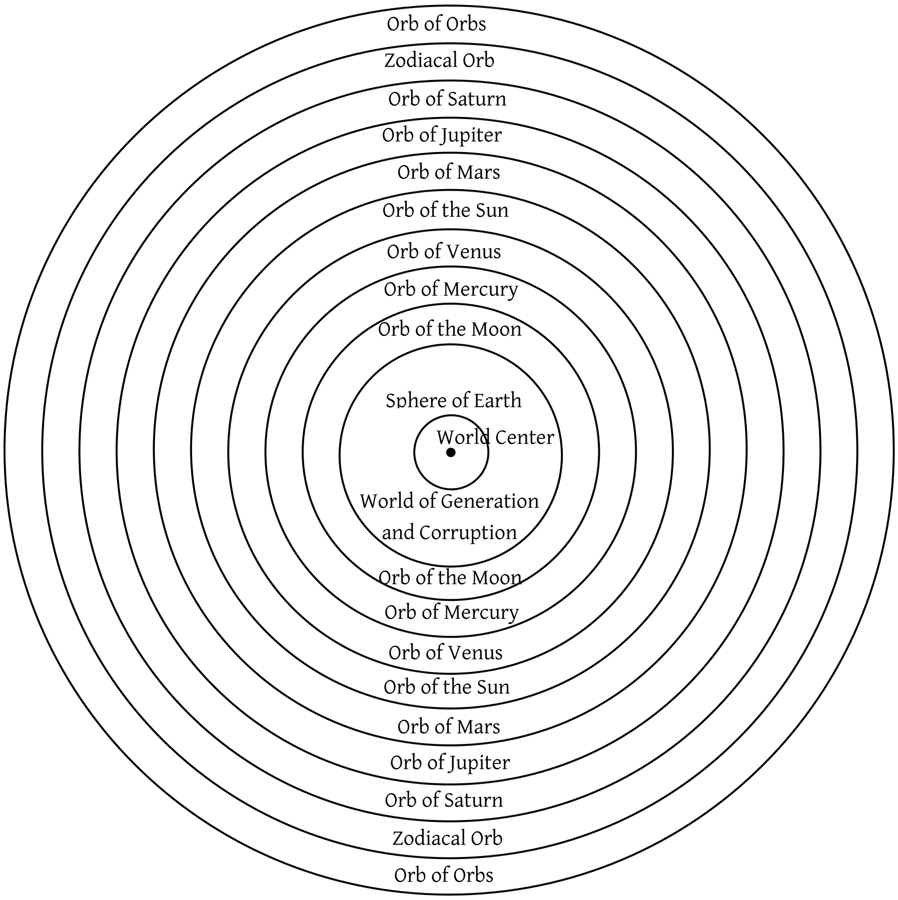
[10] As for the order: the Moon always obscures any stars with which it comes into conjunction, and no star obscures it. Mercury [obscures] Venus, Venus [obscures] Mars, Mars [obscures] Jupiter, Jupiter [obscures] Saturn, and Saturn [obscures] some of the fixed stars. There is never any difference or variation in this order. Thus, for this reason the orbs have been so arranged.

[11] The fact that the Master Abū ʿAlī [Sīnā]—may God have mercy upon him—is reported to have said, "I saw Venus as a mole on the face of the Sun," also attests to this order. Testing conjunctions with the Sun is difficult, since no star appears in its rays, except the Moon, which obscures it during solar eclipses. It has therefore been recognized that the Sun is beyond the Moon. It then became clear, based on parallax—the meaning of which will come to be known later—that the Sun is beyond Venus and below Mars, since no star has parallax less than the Sun, and Mars has no perceptible parallax. From [geometrical] demonstration, the closer anything is to the Earth the greater its parallax. It then became known that the Sun is between Mars and Venus. And this arrangement is connected with the natural order, for whatever is farther from the Earth has a larger circuit and a slower motion. Secondly, since the other wandering planets are linked with the Sun—as will become known later—and the link of the three upper planets is of one sort, the link of the two lower planets is of another,

and the link of the Moon is of yet another, then the position of the Sun in the middle of these six is closer to the natural order. The order of these eight orbs being known, the body that was posited for the purpose of moving them all with the primary motion must encompass them all, since moving an external by an internal, the thickness of the former being many, many times that of the latter, would be considered quite objectionable; and since the motion of this body is closer to simplicity than the other bodies, which are moved by its motion and have additional motions, and furthermore the sphere of the fixed stars is more simple as it has no motion other than the two [aforementioned], then placing it in a way that encompasses all [other orbs] seemed better for this reason as well. For these reasons the universal mover is called the **first orb**.

[12] Then, based on these considerations, nine orbs have been arranged in this order, starting from the top: first, the **Orb of Orbs**, which is also called the **starless orb** and the **Atlas orb**; second, the orb of the fixed stars, which is also called the **zodiacal orb**; third, the orb of Saturn; fourth, the orb of Jupiter; fifth, the orb of Mars; sixth, the orb of the Sun; seventh; the orb of Venus; eighth, the orb of Mercury; and ninth, the orb of the Moon. If, however, one starts from [our] direction, they will be in reverse, i.e., first the orb of the Moon, and ninth the Orb of Orbs.

[13] The world of generation and corruption is inside the orb of the Moon. Its order, as has become known to natural philosophers, is thus: first, the sphere of fire; second, the sphere of air; third, the sphere of water; and fourth, the sphere of earth. Earth and water are mixed together, since water's encompassment of earth is incomplete, as will be explained later. The Earth is the center of the World and in the middle of the bodies; an illustration of these bodies in their encompassment is as set forth [in the following]—God is all-knowing:



[Figure 1]

CHAPTER TWO

An Exposition of the Primary and Secondary Motions, and the Names of the Great Circles

[1] In the introduction we said that when a sphere has rotational motion, there results an equator, two poles and an axis, and that the equator is one of the great circles. Now we say that two different rotational motions in one sphere cannot be perceived, unless they are around either different poles or different centers; so that, for example: [1] the two poles of one motion are different from the two poles of the other

motion in such a way that one axis intersects the [other] axis at the center, and one equator intersects the other in two places; or [2] one motion is around one center and the other is around another center, and this necessitates that that sphere becomes two spheres, the equator of one lying in the plane of the equator of the other, the center [of one] outside the center of the other, and the [two] axes parallel. If, however, the [two] centers are one and the poles are at the selfsame two points, only one motion can be perceived, and that motion amounts either to the sum of the two motions in question if both are going in the same direction, or to the difference between the faster and the slower if they are going in different directions.

[2] All celestial motions, relative to one another, are of these two types only. As for one body moving another body inside it, if the difference between the two motions is of the first type, then the poles of the internal sphere are inseparable from two specific points on the external sphere. Then, by the motion of the external sphere, those two points are displaced, and the whole [internal] sphere moves because of the displacement of its poles. If the difference is of the second type, such that the internal sphere and its center is part of the external sphere, then the former is moved by the motion of the latter.

[3] However, the moving by the orb of the sphere of aether, which is the sphere of fire, this being known from the motion of comets when, upon reaching that place, they move along with the orb—is neither of these two types. Rather, it is by way of a conformity to its own place. In other words, when [aether's] place—which is the concave surface of the orb—moves, it will move by way of conformity, since everything firmly fixed in a place will cling and adhere to it. The farther away it is [from the orb], the weaker its motion becomes, until it ceases altogether.

[4] After this introduction, we say that when the Orb of Orbs moves with the primary motion, it inevitably has an equator and two poles. Its equator is called the **equinoctial [celestial equator]**, and its two poles are called the poles of the primary motion and also the poles of the equinoctial. The reason this circle is called the equinoctial is that when the Sun reaches this circle, day and night are of equal length. Due to this motion, every point on the surface of the sphere produces a circuit parallel to the equinoctial. These circuits are called **day-circles**. These circles and circuits must be imagined on all the orbs of the stars, since they are all subject to the primary motion.

[5] When the zodiacal orb and its motion are considered, there is produced another equator that intersects the equinoctial at two places. That equator is called the **zodiacal orb**, or the **zodiacal equator**. Its two poles, which are on opposite sides of the two equinoctial poles, are called the two poles of the zodiacal equator. The planes of the equinoctial and the zodiacal equators intersect each other at acute and obtuse angles, and the maximum distance between the two equators is the maximum distance between the two poles. This is called the **maximum** or **total obliquity**.

[6] Every star has a circuit parallel to the zodiacal equator; [collectively] they are called the parallels of latitude. When a circle is imagined that passes through all four poles—i.e., the two equinoctial poles and the two poles of the zodiacal orb—it will inevitably pass through the maximum distance between the two equators. This circle is called the **solstitial colure** [lit., that passing through the four poles], and its two poles are the two points of intersection between the equinoctial and the zodiacal orb. The plane of this circle is at right angles to the planes of the first two circles. The zodiacal orb is divided into four equal parts by the equinoctial and this circle: a vernal quarter and a summer quarter—and these two quarters are north of the equinoctial equator—and an autumnal quarter and a winter quarter—and these

two quarters are to the south [of the equinoctial equator].

[7] The two points of intersection between the equinoctial and the zodiacal orb are called the **two equinox points**: the one the Sun comes to in going toward the north is called the **vernal equinox**; the other is the **autumnal equinox**. The two points of intersection between the solstitial colure and the zodiacal orb are called the **two solstice points**: the northern being summer and the southern winter.

[8] The arc on the solstitial colure falling between the two equators is called the **obliquity** and is equal to the arc between the two poles. That which is between the equator of one and the pole of the other is called the **complement of the obliquity**. If one takes a circle to have 360 parts, the total obliquity, according to the observation of Battānī and other moderns, is 23°35′, with a complement of 66°25′. In the observations made during the reign of Maʾmūn, they found 23°33′. Ptolemy found it to be 23°52′, and prior to him it was taken to be 24°0′.

[9] A group of moderns has said that since these variations are proportionally decreasing, it appears that the two equators are coming closer to each other. If this is so, then one of two things will happen: either they will continue to proceed toward each other until they reach a point at which they coincide, and day and night will be equal throughout the World, and afterwards they will pass each other, the northern half of the zodiacal equator becoming southern and the southern half northern; or else, their proceeding toward each other will have a limit and when it is reached they will move away from each other again until another limit is reached. In either case, another body must be assumed to be the principle of this motion.

[10] When one takes a [discrete] part on the orb and it is desired to find the distance of that part from the equinoctial equator, a circle should be imagined that passes through that part and also the two poles of the equinoctial equator, its plane thus being perpendicular to the equinoc-

tial equator. This circle is called the **declination circle**. What is on this circle between that part and the equinoctial equator is the distance of that part from the equinoctial equator. When this circle is assumed for the parts of the zodiacal equator, what lies on this circle between the zodiacal equator and the equinoctial equator is called the **first declination**. This circle is unique in type but infinite in individuation when one assumes one [circle] for every part, in contrast with the first three circles, which never vary anywhere in the World nor with respect to particular parts.

[11] If it is desired to relate a certain part to the zodiacal equator with respect to distance, then the circle must be considered in a way that passes through that part and through the two poles of the zodiacal equator. It is called the **circle of latitude**. What lies between that part and the zodiacal equator on this circle is called the latitude of that part, and what lies between the zodiacal and the equinoctial equators on this circle is called the **second declination**. The relation of this circle to the zodiacal equator is like the relation of the declination circle to the equinoctial equator, and the condition of this circle is like that of the declination circle, infinite in individuation but unique in type. The latitude of stars can be known from this circle. This latitude is cited with respect to longitude. For true position, longitude is needed. Longitude is the arc on the zodiacal equator between the vernal equinox point and where this [latitude] circle intersects the zodiacal equator. If a star has no latitude, the longitude is between the vernal equinox point and the center of the star. The position with respect to the zodiacal equator of a star having latitude is the point of intersection of the zodiacal equator with its latitude circle.

[12] If six circles of latitude are conceived such that the zodiacal orb is divided into twelve equal segments, like the segments of a melon, the intersection of these circles with one another being of course at the two poles of the zodiacal orb—one of these circles being the solstitial

colure and another passing through the two equinox points—then the [resulting] divisions are called the **twelve signs of the zodiac**. Each sign has a 30° longitudinal length and a 180° latitudinal width from pole to pole. On this basis, a star that is away from the zodiacal equator is said to be “in” a sign when it is in one of these divisions.

[13] These then are five of the great circles imagined on the orb that are not based upon a relation to terrestrial locations. Now when terrestrial locations are taken into consideration, the circle that separates the visible half of the orb from the invisible half is called the **horizon circle**. It has two poles: the one that is up is called the **zenith**, and the one opposite it below the Earth is called the **nadir**. Of the circles parallel to this circle that pass through loci on the orb, those that are above are called **almucantars of altitude** and those that are below are called **almucantars of depression**. If another circle is conceived passing through the two poles of the celestial equator and the two poles of the horizon circle, no doubt also at right angles both to the celestial equator and the horizon, that circle is called the **meridian circle**, and it divides the World into eastern and western hemispheres. This circle has two poles, one the east point and the other the west point. The stars reach this circle twice in every revolution: once at the midpoint of the period of visibility and once at the midpoint of the period of invisibility. They also reach the horizon circle twice: once at the time of rising and once at the time of setting. If another circle is conceived passing through the two poles of the meridian and the two poles of the horizon, no doubt also being at right angles to both the meridian and horizon, that circle is called the **east-west circle**, or circle of the **initial azimuth [prime vertical]**. It has two poles, one the north point and the other the south point.

[14] By means of these three circles, the [celestial] orb is divided into eight parts, four above and four below. One of the four is between east and north; the second between west and north; the third between west

and south; and the fourth between east and south. Each of these three circles is unique in type but numerous in individuation, according to given localities on the Earth. If one takes a celestial locus and desires to know its distance from the horizon, a circle should be taken that passes through it, namely, that locus, and the two poles of the horizon, i.e., the zenith and its opposite. This circle is called an **altitude circle**. What is between the horizon and that locus on this circle is called the [arc of] altitude. When the star reaches the meridian, this circle will coincide with the meridian; what is between this circle and the initial azimuth on the horizon circle is called the **azimuth** of the star. If the star has no azimuth, its altitude circle will be the initial azimuth. Considering the celestial loci, there are also numerous altitude circles.

[15] From this discussion, nine great circles are now known: [1] the equinoctial; [2] the zodiacal orb; [3] the solstitial colure; [4] the declination circle; [5] the latitude circle; [6] the horizon circle; [7] the meridian circle; [8] the initial azimuth circle; and [9] the altitude circle. This then is the intention of this chapter.

CHAPTER THREE
An Exposition of the Circumstances of the Eighth Orb and the Fixed Stars

[1] Every star, aside from the Sun and Moon (lit., the luminaries) and the five vacillating planets, is reckoned one of the fixed stars, and all of these stars are fixed on the eighth orb, which we call the orb of the fixed stars and the orb of the zodiacal signs. They are called "fixed" due to the slowness of their movement, i.e., fixed compared to the planets, or due to the immutability of their positions [in the sky] and with respect to one another, since these stars always maintain their latitude and move only in longitude. Their longitudinal movements were not perceived by the ancients, who reckoned them as fixed; but later, dur-

ing the time of Ptolemy and Menelaus, their movement was perceived, and it was said that every hundred solar years they move one degree. When the moderns compared their observations with those of the ancients, [it became known] they move one degree every sixty-six solar years. The view of this latter group has been accepted.

[2] Each of these stars has two circuits: one for primary motion, which is one of the diurnal circuits; and the other one for the second motion, which is one of the latitude circuits. The latitude circuit never becomes larger and smaller—because the latitude of these stars is never-changing—but the diurnal circuit does become larger and smaller. Therefore, every star that has no latitude, by being on the zodiacal equator, reaches the equinoctial equator twice in every revolution, [and therefore] for about 12,000 years it is in the north, and for about 12,000 years in the south. A star whose latitude is less than the total obliquity also reaches the equinoctial equator twice, but the duration of its stay in the north and south is unequal. One whose latitude is equal to the total obliquity reaches the equinoctial equator once in every revolution, but does not shift from side to side. One whose latitude is greater than the total obliquity never reaches the equinoctial equator. One whose latitude is equal to the complement of the total obliquity reaches the pole of the equinoctial equator once in every revolution and then has no diurnal circuit.

[3] Now it has become known from this that the positions of the fixed stars with respect to the horizon of any given locality fluctuate. Sometimes [a star] that never rises will come to rise when its colatitude is greater than the difference between the latitude of the locality and the total obliquity; and sometimes a permanently visible one becomes invisible under the same condition. Sometimes [a star] that used to transit the zenith will move away from the zenith, and sometimes one that did not reach the zenith will transit it. After one complete revolution, they all return to their initial positions. When Polaris [α Ursae

Minoris], a star in *Banāt al-naʿsh-i ṣughrā* [Ursa Minor] whose latitude is close to the complement of the total obliquity, reaches the first of Cancer—which will be after 870 years from our current date—it will fall closer to the north [celestial] pole, and then observation of the pole will be easy, since its altitude will be equal to the latitudes of the localities.

[4] There are too many fixed stars to enumerate, as can be easily perceived. However, the larger ones, on which the gaze can be fixed, have been counted. They are said to be of six magnitudes. The largest are of the first magnitude, then the second magnitude, and so on to the sixth. Those of the sixth magnitude are one sixth of those of the first magnitude, and those of the fifth magnitude are one third and so on. All the counted and observed stars are 1,022: first magnitude, 15; second magnitude, 45 stars; third magnitude, 208 stars; fourth magnitude, 474 stars; fifth magnitude, 217 stars; and sixth magnitude, 49 stars. There are nine obscure stars, which Ptolemy calls "faint."[1] There are five nebulae which are like wisps of cloud. There are three other stars similar to faint stars, which are called Dhuʾāba or Ḍafīra; they are not included among the stars.

[5] For convenience, constellations have been imagined out of several stars, so that they can be easily specified in such a way that a given star can be said to be "on the hand" or "on the head" of a given constellation. There are 21 such constellations to the north [of the zodiacal equator], 12 on the zodiacal equator, and 15 to the south [of the zodiacal equator]. Some of these stars are within the constellations, while others are outside the constellations. The constellations are as

1. In the *Almagest*, these are called amauros, which Toomer translates as faint (Claudius Ptolemy, *Ptolemy's Almagest*, trans. G.J. Toomer (London: Duckworth,1984), 16).

follows:[1]

[6] Northern Constellations:

(1) Ursa Minor, contains seven stars, one outside;
(2) Ursa Major, contains twenty-seven stars, seven[2] outside;
(3) Draco, contains thirty-one stars;
(4) Cepheus, contains eleven stars, two outside;
(5) Boötes, contains twenty-two stars, one outside;
(6) Corona Borealis, contains eight stars;
(7) Hercules, contains twenty-nine[3] stars, one outside;
(8) Lyra, contains ten stars;
(9) Cygnus, contains seventeen stars, two outside;
(10) Cassiopeia, contains thirteen stars;
(11) Perseus, contains twenty-six stars, three outside;
(12) Auriga, contains fourteen stars;
(13) Ophiuchus, contains twenty-four stars, five outside;
(14) Serpens, eighteen stars;
(15) Sagitta, five stars;
(16) Aquila, nine stars, six outside;
(17) Delphinus, ten stars;
(18) Equuleus, four stars;
(19) Pegasus, twenty stars;

1. There are problems with the numbers for a few constellations (noted in the following footnotes). These are discrepancies with what one finds in the *Almagest* that result in totals different from those Ṭūsī gives. We have left these "mistakes" in both the edition and the translation, since they are attested by our most reliable manuscript witnesses. As far as we can ascertain, Ṭūsī here relied on a manuscript of Kharaqī's *Muntahá* that may have been the origin of these mistakes. However, in his recension of the *Almagest* (completed in 644 H/1247 CE), he has the "correct" numbers. One hopes further research will clarify the situation.

2. Eight in the *Almagest* (343).

3. Twenty-eight in the *Almagest* (349).

(20) Andromeda, twenty-two[1] stars;
(21) Triangulum, four stars.

[7] Constellations of the zodiacal equator:

(1) Aries, thirteen stars, five stars outside;
(2) Taurus, thirty-three[2] stars, eleven stars outside;
(3) Gemini, eighteen stars, seven stars outside;
(4) Cancer, seven[3] stars, four stars outside;
(5) Leo, twenty-seven stars, eight stars outside;
(6) Virgo, twenty-six stars, six stars outside;
(7) Libra, eight stars, nine stars outside;
(8) Scorpius, twenty-one stars, three stars outside constellation;
(9) Sagittarius [*Rāmī*] which is called *Qaws*, thirty-one stars;
(10) Capricorn, twenty-eight stars;
(11) Aquarius [*Sākib al-māʾ*] which is also called *Dalw*, forty-two stars, three stars outside constellation;
(12) Pisces, thirty-four stars, four stars outside.

[8] Southern Constellations:

(1) Cetus, twenty-two stars;
(2) Orion, thirty-eight stars;
(3) Eridanus, thirty-four stars;
(4) Lepus, twelve stars;
(5) Canis Major, eighteen stars, eleven stars outside;
(6) Canis Minor, two stars;
(7) Argo, forty-five stars;

1. Twenty-three in the *Almagest* (360).

2. Thirty-two in the *Almagest* (363).

3. Nine in the *Almagest* (366).

(8) Hydra, twenty-five stars[1];
(9) Crater, seven stars;
(10) Corvus, seventeen[2] stars;
(11) Centaurus, thirty-seven stars,
(12) Lupus, nineteen stars;
(13) Ara, seven stars;
(14) Corona Australis, thirteen stars;
(15) Piscis Austrinus, eleven stars, six stars outside.

[9] Thus the sum of the stars of the northern constellations is 360. The sum of the stars of the zodiacal constellations is 346, and of the southern constellations, 316.

[10] The names of the zodiacal constellations have been given to the twelve divisions of the zodiacal equator—i.e., the Twelve Signs—because, by chance, they are equivalent to these divisions. As these stars move from those places, they say, e.g., that the stars of Aries have entered Taurus. Nonetheless, the name of Aries—which is the first of the zodiacal divisions and is contiguous with the point of the vernal equinox—remains fixed. If Aries were to be called Pisces, it would make no difference because the important thing is the meaning, not the name. The twelve signs, to which the six latitudinal circles give rise, primarily for the zodiacal sphere and secondarily for all the spheres of the stars, have no connection with the bodies of the stars. Since all parts of the zodiacal sphere move with the primary motion, its two poles move around the two poles of the equinoctial equator. When it reaches the greatest and lowest altitude of its path, i.e., on the meridian, the solstitial colure coincides with the circle of the meridian.

1. *Almagest* adds 2 outside (393).

2. Seven in the *Almagest* (394).

[11] The Arabs divide these constellations in a different way. They imagine twenty-eight stations close to the equator, and these they call the "mansions of the Moon," since a complete revolution of the Moon is accomplished in approximately twenty-eight nights. The names of the mansions are: (1) Sharaṭayn; (2) Buṭayn; (3) Thurayyā; (4) Dabarān; (5) Haqʿa; (6) Hanʿa; (7) Dhirāʿ; (8) Nathra; (9) Ṭarf; (10) Jabha; (11) Zubra; (12) Ṣarfa; (13) ʿAwwā; (14) Simāk; (15) Ghafr; (16) Zubānā; (17) Iklīl; (18) Qalb; (19) Shawla; (20) Naʿāʾim; (21) Balda; (22) Saʿd-i dhābiḥ; (23) Saʿd-i bulaʿ; (24) Saʿd-i suʿūd; (25) Saʿd-i akhbiya; (26) Fargh muqaddam; (27) Fargh muʾakhkhar; (28) Rashā.

[12] Every $2+\frac{1}{3}$ of these mansions is a zodiacal sign, and the relation of the stars of the mansions to the mansions is exactly the relation of the stars of the constellations to the constellations. Anyone who wants to learn about the fixed stars in detail should consult books on this science, which is an art unto itself. The best book done on this subject is the *Ṣuwar al-kawākib* by ʿAbd al-Raḥmān Ṣūfī. This is all we intended to say about the fixed stars—with God is success.

CHAPTER FOUR
An Exposition of the Orbs and the Motions of the Sun

[1] When the circumstances and motions of the Sun were observed, it was found to be moving from west to east by its own proper motion such that it makes one revolution in one solar year. However, it does not cut equal arcs on the [zodiacal] orb in equal periods of time but rather moves faster in one half of the [zodiacal] orb and slower in the other half. Considering the uniformity of circumstances that must pertain in celestial matters, this fastness and slowness in speed is possible by either one of two ways.

[2] The first is that the Sun's body moves on the circumference of an orb whose center is eccentric to the center of the World, while it en-

compasses the Earth, such that when the Sun is nearer to the Earth in one half of that [eccentric] orb and farther from it in the other half, equal arcs on that orb seem unequal with respect to the center of the World. Therefore, in one half, faster motion is produced and in the other half, slower motion. Such an orb is called an eccentric.

[3] The second is that the body of the Sun moves on the circumference of an orb whose center is not the center of the World; its circumference does not enclose the Earth, but rather is a small orb situated in the thickness of a body that does encompass the Earth. That body has uniform motion. Now there is no doubt that the motion of the Sun on the circumference of that small orb, in one half, will be in the same direction as the motion of the encompassing body with respect to the Earth and in the opposite [direction] in the other half. Therefore, in the half going in the same direction, the motion of the Sun appears to be composed of the sum of the two motions and is faster, while in the other half is composed of the excess of the motion of the encompassing body over the motion of the small orb, and [consequently] appears slower. Such a [small] orb is called an epicycle.

[4] Ptolemy chose the eccentric orb for the Sun because it is simpler, since from the motion of the Sun on the circumference of the epicycle, and the motion of the epicycle on the circumference of the orb that carries it, a circuit will be produced for the Sun, eccentric to the center of the World. Therefore, by positing the epicycle it is necessary to posit an eccentric too, but positing the eccentric does not necessitate positing the epicycle. This being the case, the eccentric is simpler and more appropriate to be posited.

[5] The result of this consideration is that the Sun has two orbs, one of which has the same center as the Earth and two parallel surfaces encompassing it: the upper surface, which is called the convex, is tangent to the lower surface of the orb of Mars; the lower surface, which

is called the concave, is tangent to [the upper surface of] the orb of Venus. The equator and two poles of this orb are in the plane coinciding with the equator and poles of the zodiacal orb. This orb is called the parecliptic orb, meaning [it has been] likened to the zodiacal [ecliptic] orb.

[6] The second orb is one that encompasses the Earth. Its center is eccentric to the center of the World, and it is within the thickness of the parecliptic orb, so that its convex surface is tangent to the convex of the parecliptic at one common point, and its concave surface is tangent to the concave of the parecliptic, also at one common point directly opposite the first point. The equator of this orb is in the same plane as the equator of the first orb, and its axis is parallel to the axis of that [first one]. This orb is called the eccentric orb.

[7] The Sun [itself] is a solid spherical body in the thickness of this eccentric orb, such that its convexity is tangent to both surfaces of the eccentric orb. The eccentric moves with the mean motion of the Sun—which is 0;59,8 daily—and carries the Sun along with itself. Therefore, in one half, i.e., the upper half, the amounts of the arcs appear smaller, hence it cuts smaller [arcs] from the zodiacal orb than the amount of the mean motion, and the motion is slower, and in the lower half vice versa.

[8] The midpoint of the slow days, which is also the farthest point from the center of the World, is called the apogee, and the farthest distance. The point that is directly opposite it, which is the midpoint of the fast days, is the nearest point to the center of the World; it is called the nearest distance and the perigee. Ptolemy did not find any motion for the apogee and perigee and stated that the apogee is at 5;30 in Gemini, and it is fixed. However, the moderns have found it moving with the motion of the fixed stars. So they attributed this motion to the parecliptic orb, so that when it moves with this motion it carries all parts of

the eccentric orb with it; thus the apogee and perigee also move with this motion.

[9] The mean distance of the Sun is where the two lines extending from the center of the World and the center of the eccentric to it are equal. This occurs at two points on either side of the apogee.

[10] The two bodies left from the parecliptic after removing the eccentric orb from it are called the complements.

[11] Since the Sun moves along the equator of the eccentric orb and this equator is in the plane of the zodiacal orb, the Sun always adheres to the equator of the zodiacal orb and has no latitude in any direction.

[12] When two lines are extended from the two centers—i.e., the center of the parecliptic, which is the center of the World, and the center of the eccentric—to the body of the Sun, and from there to the equator of the parecliptic orb, the position of the Sun with respect to the center of the World is obviously other than its position with respect to the eccentric center. This difference is called the equation of the Sun, and the angle occurring at the Sun's body by these two lines is called the angle of the equation.

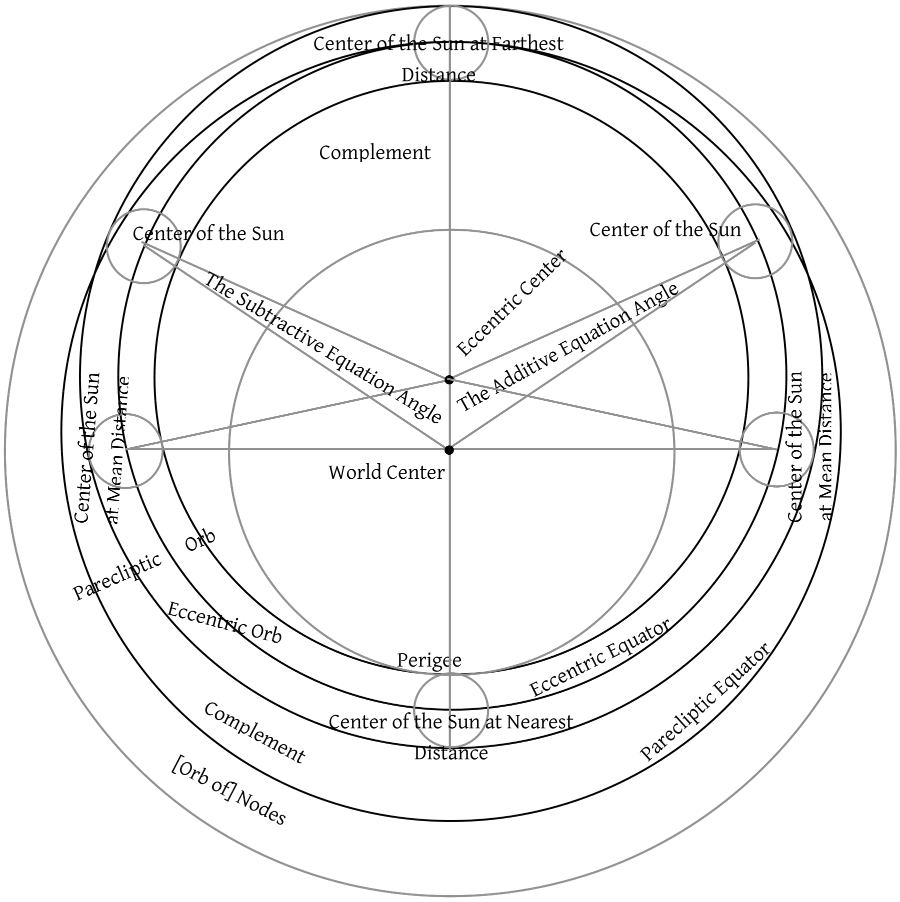
[13] The mean position of the Sun is [determined] with respect to the eccentric center, and the true position with respect to the World center. The mean Sun is an arc on the parecliptic equator between the vernal equinox point and the endpoint of a line extending from the eccentric center to the center of the Sun's body. When the point of farthest distance is taken as the initial point of this arc, this will be the arc of the Sun's center. The adjusted apogee or farthest distance is an arc between the beginning of Aries and the point of farthest distance. The true position is an arc along the parecliptic orb between the beginning of Aries and the endpoint of the line extending from the World center to the center of the Sun's body.

[14] Since the equation, which is the difference between the mean and the true position, arises from the difference between the two lines extended from the two centers to the body of the Sun, and since the endpoint of the line extended from the World center is always closer to the apogee, then while the Sun is between the apogee and perigee, the equation is subtractive from the mean and additive in the other half. At the apogee and perigee there is no equation, since both lines coincide.

[15] The distance between the two centers has been found to be 2;4,45 by observation—the radius of the eccentric being 60. This amount is used to find the equation. [The distance] is found to be 2;1—the radius of the parecliptic being 60. This amount is used to find the distance of the Sun from the Earth.

[16] It has been established then that the Sun has two orbs and two motions. Practitioners of geometry are satisfied with two circles: one being the equator of the eccentric and the other the equator of the parecliptic, on the condition that the eccentric equator passes through the center of the Sun and the parecliptic equator is tangent to it. This is an explanation of the configuration of the Sun's orbs. Here is its illustration.

[17] The two circles that are in black represent those with which the practitioners of the science of geometry are satisfied; they call them the parecliptic orb and the eccentric orb.



[Figure 1]

CHAPTER FIVE
An Exposition of the Orbs and Longitudinal Motions of the Moon

[1] When the situation of the Moon is considered, a faster and a slower motion will also be found in the course of its movement, and [also] an approach toward the Earth and a receding from it. [This] approach and receding of [the Moon] is determined by parallax. However, its situation in these positions is opposite to the situation of the Sun, since the Sun always has a slow speed when it is situated farther from the

Earth and has a fast speed when it is situated nearer. The Moon may sometimes be at the farthest distance with a fast speed, or again may sometimes be at the farthest distance with a slow speed. [Likewise] it may sometimes be at the nearest distance with a fast speed, or may be again at the nearest distance with a slow speed.

[2] Its farthest distance from the Earth is always at the time of conjunction and opposition, and its nearest distance is during the two quadratures with the Sun. Its circuit does not correspond to the Sun's, but rather is sometimes north of the Sun's circuit—i.e., of the zodiacal equator—and sometimes south. The two points of intersection between these two circuits move from east to west. Therefore, due to these irregularities, four solid orbs and four uniform motions were posited so that these matters could be ordered by compounding these bodies and motions.

[3] As for the orbs, the first orb is an orb whose center is the center of the World and whose two poles and equator are congruent with the poles and equator of the zodiacal orb and in the same plane. Its convexity is contiguous to the concavity of Mercury's orb, and its concavity is contiguous to the second of the Moon's orbs. This orb is called the parecliptic.

[4] The second orb is an orb whose center is also the center of the World, and whose equator is not in the plane of the zodiacal orb, but rather one half [of it] is north of that equator [of the zodiacal orb] and one half south of it, like what we have said in the case of the equinoctial and the zodiacal orb. Its two poles are on opposite sides with respect to the two poles of the parecliptic. Its convexity is tangent to the concavity of the first orb, and its concavity is tangent to the world of generation and corruption. This orb is called the inclined orb.

[5] The third orb is an eccentric orb in the thickness of the inclined orb, in the same way that the Sun's eccentric is in the thickness of its

parecliptic, i.e., its equator is in the plane of the inclined orb's equator, its convexity is tangent to the [latter's] convexity at a single point, and its concavity is tangent to the [latter's] concavity at a single point.

[6] The fourth orb is an epicycle orb, embedded in the thickness of the eccentric orb, as has been explained previously, such that its convexity is tangent to both surfaces of the eccentric orb at two points. The Moon's body is situated in the epicycle orb like a jewel in a ring, such that the convexity of its spherical body is tangent to the convexity of the epicycle orb at one common point.

[7] As for the motions: the first motion is the motion of the parecliptic orb in the counter-sequence of the zodiacal signs, 0;3 per day, and all the remaining orbs are carried with this motion. Since this motion is perceived at the intersection of the equator of the parecliptic and the inclined, it is called the motion of the nodes (*jawzahar*), because the two intersections are called the *jawzahar*. The parecliptic orb is called the *jawzahar* orb. The intersection of these two equators is conceived on the inclined orb, just as we have said concerning the intersection of the equinoctial and the zodiacal orb. Of these two intersections, the one that the Moon upon reaching it becomes northerly with respect to the Sun's circuit is called the northern crossing point or the head; and the other, which is directly opposite it, is called the southern crossing point, or tail. The maximum obliquity between these two equators in either direction is 5;0, and this is the maximum latitude of the Moon.

[8] The second motion is that of the inclined orb, which is also in the counter-sequence of the zodiacal signs, 11;9 per day. Because this motion is perceived at the apogee and perigee of the eccentric [orb], it is called the motion of the apogee and the farthest distance. The eccentric orb and epicycle orb move with this motion as well.

[9] The third motion is that of the eccentric orb moving in the sequence of the zodiacal signs, 24;23 per day. Because this motion is

perceptible at the epicycle center, it is called the motion of the center. The epicycle moves with this motion.

[10] The fourth motion is that of the epicycle orb, 13;4 per day along its circumference, such that in the upper half it is in the counter-sequence of the zodiacal signs and in the lower half it is in the sequence of the zodiacal signs. Since this motion is perceptible in the body of the Moon, it is called the proper motion.

[11] The farthest position of the epicycle orb with respect to the center of the World is the apex, and its closest position is the [epicyclic] perigee.

[12] The equator of the epicycle orb is always in the plane of the equator of the eccentric orb, and the equator of the eccentric orb is in the plane of the equator of the inclined orb. The eccentric orb is also called the deferent orb of the epicycle.

[13] Therefore, due to the motion of the epicycle orb and the displacement of the Moon's body with it, fastness and slowness are produced in the Moon's motion. Because in the upper half, where the motion of the epicycle is in the counter-sequence [of the zodiacal signs], the [combined] motion [which is] in the sequence [of the zodiacal signs] is slower by the amount of that motion subtracted from [the sequential motion]. In the lower half it is faster, because the two motions agree [in direction], in the amount of the addition of this motion to it. Because of the motion of the eccentric orb and the displacement of the epicycle by it, the Moon approaches and recedes from the Earth, so that it can be fast or slow whether far or near.

[14] Since the parecliptic orb moves the other orbs in the counter-sequence [of the zodiacal signs], the two nodes move in the counter-sequence. Since the Moon is in the plane of the inclined orb, and the plane of the inclined orb is slanted with respect to the pareclip-tic plane, the Moon gains latitude on the zodiacal orb to the north and

south. Therefore, its circuit is different from that of the Sun and intersects the Sun's circuit at two places. The center of the lunar epicycle at the time of conjunction and opposition is always at the deferent orb's apogee. Therefore, since the parecliptic moves 0;3 daily in the counter-sequence, and the inclined 11;9, also in the counter-sequence, the apogee moves away from its initial position by the sum of these two amounts: 11;12. And since the center of the epicycle moves 24;23 in the sequence, the apogee moves away from the center of the epicycle by this [same] amount. Therefore, the distance of the center of the epicycle from its initial position amounts to 13;11, and this is the mean motion of the Moon, since the displacement of the Moon in the signs appears to be this much.

[15] The Sun moves 0;59 [degrees per day] in the sequence from its initial position; [if] we subtract this amount from the mean motion [of the Moon], 12;12 remains, which is the distance of the center of the epicycle from the Sun. If we add this same amount to the distance of the apogee from its initial position, it becomes 12;12, which is the distance of the apogee from the Sun, and it is equal to the distance of the center of the epicycle from the Sun. Therefore, the Sun is always at the midpoint between the apogee and the center of the epicycle. The distance of the apogee from the center of the epicycle is twice the distance of each one from the Sun. For this reason, the motion of center of the epicycle is called the double elongation.

[16] In this manner, the apogee moves in the counter-sequence and the center of the epicycle moves in the sequence, so that when each one has completed half a circuit, they will be together at opposition of the Sun. Then, in opposition, the center of the epicycle is at apogee once more, and [the two] again pass away from each other until they reach each other in conjunction. At the quadratures, the apogee and the center of epicycle stand opposite one another. So, the center of the epicycle is always at apogee in conjunction and opposition, and at

perigee in quadratures.

[17] From these configurations, there result three anomalies for the Moon: the first [anomaly] is the difference that arises from two lines extending from the center of the World, one to the center of the epicycle and the other to the center of the body of the Moon. This difference is called the independent equation, and in some *zīj*es the second equation. This is on account of the proper [motion], and its maximum reaches the amount of the radius of the epicycle orb. The amount of the radius of the epicycle orb, assuming the radius of the inclined orb as 60, is 5;15. When the Moon is at epicyclic apex or perigee, the two above-mentioned lines coincide with each other and there is then no equation. Since its motion from apex is in the counter-sequence, during the time that the Moon is between apex and perigee, the equation is subtractive, and additive in the other half.

[18] The second anomaly arises from the approach of the epicycle orb toward and its receding from the center of the World. Accordingly, whenever the center of the epicycle is at the apogee of the deferent, the radius of the epicycle appears less than when at the deferent's perigee. This difference is called the anomaly of the farthest and nearest distance. This difference being additive and subtractive follows the independent equation's being additive and subtractive, since [this anomaly] is actually dependent upon it.

[19] The third anomaly results from the proper motion on account of the mean apex and perigee of the epicycle orb not being the apparent apex and perigee, which are [defined] with respect to the center of the World, i.e., the diameter of the epicycle that passes through the apex and perigee. It is not aligned with the center of the eccentric, nor with the center of the World, except when the center of the epicycle is at apogee or perigee of the deferent orb. However, it is always aligned with a point whose distance from the center of the World, in the direc-

tion of the perigee, is equal to the distance of the center of the deferent from the center of the World. The amount of distance between these two centers, assuming the radius of the deferent as 60, is 12;30. This [amount] is used to calculate this anomaly. By assuming the radius of the inclined [orb] as 60, it is 10;19; and this [amount] is used to find the distance of the Moon from the Earth.

[20] Then, when the center of the epicycle is at apogee or perigee, this third anomaly is imperceptible, because the diameter of the epicycle is aligned with the center of the World as well as the center of the eccentric, and prosneusis point. However, when it is between apogee and perigee, the end of a line extending from the prosneusis [i.e., alignment] point to the center of the epicycle and thence to the equator of the inclined [orb] is closer to the apogee than is the end of a line extending from the center of the World; [so] the apparent apex is farther from the apogee. Therefore, the distance between the two apices must be added to the proper motion, the initial position of which is taken from the mean apex, to obtain the adjusted proper motion. In the other half, it must be subtracted. This difference is called the equation of proper motion, or the first equation. The angle occurring from these two lines at the center of epicycle is called the angle of the first equation; similarly, the angle occurring from the two lines extending from the center of the World to the center of the epicycle and to the body of the Moon is called the angle of the second equation. The mean distance of the Moon in the apogee orb is where two lines extending from the center of the World and the center of the eccentric [to the center of epicycle] are equal—as we have said for the Sun.

[21] Thus, when four orbs and four motions are posited for the Moon, this irregularity will be resolved. The orbs: the parecliptic orb; the inclined orb; the deferent orb; and the epicycle orb. Instead of orbs, the practitioners of this discipline posit circles, which are the equators of these orbs, and call them by these [same] names—as has been said

for the Sun. Thus, the parecliptic and inclined are two intersecting circles with equal radii; and the deferent is an eccentric circle that passes through the center of the epicycle and is tangent to the inclined [orb]; and the epicycle circle is the equator of the epicycle orb. A circle is also produced by the circuit of the center of the eccentric around the center of the World, which is called the deferent of the center of the deferent orb.

[22] The motions are: the motion of the nodes; the motion of the farthest distance; the motion of the center, which is called the motion of the double elongation; and the proper motion. What arises from these motions is the mean motion. The compound non-uniform motions are the adjusted proper motion and the motion of the true position.

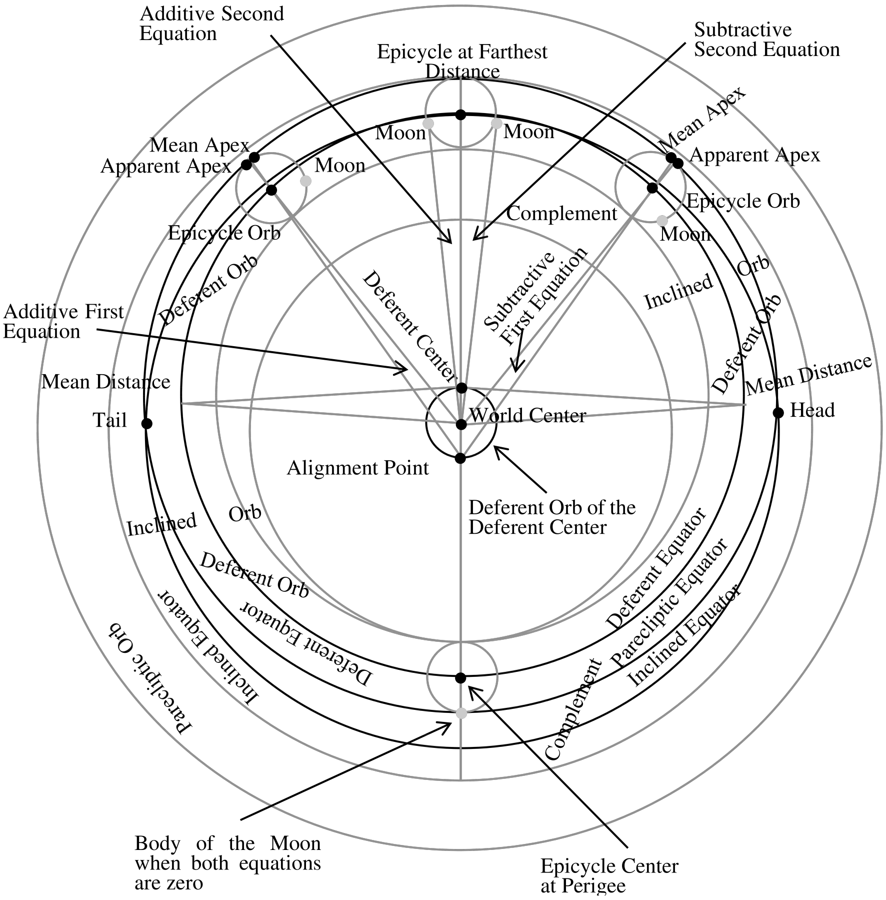
[23] Practitioners of the discipline have said that the motion of the fixed stars, which arises from the eighth orb, also pertains to the Moon but, because of [the Moon's] fast motions, it is not perceptible. This caveat they have made is utter nonsense, since, from the time of Battānī's observation, which is [fairly] close to us, the fixed stars have moved 5 degrees. If such a difference had befallen the Moon's motion, think what it would be like! If there were a 20-minute difference, for example, in the true position of the Moon, there would be such discrepancies in the lunar and solar eclipses, etc. that they could not be represented. [Now] the reason that the motion of the fixed stars is imperceptible [in the Moon] is what we have said, that two different [circular] motions in one sphere, around one equator and two given poles, are indistinguishable—rather, one motion can be perceived from the combination [of the two]. Since the motion of the nodes of the Moon's orbs is around the equator of the zodiacal orb and its poles, the motion of the fixed stars combined with that motion cannot be distinguished. Therefore, the motion of the nodes is in fact more than this perceived amount by the amount of the fixed stars' motion; thus, on account of the difference between these two motions, this amount, which has

been found by observation, has come to be perceived. This being so, there are five uniform motions.

[24] [However,] a doubt arises from the foregoing account. This [doubt] is that from the motion of the center of the epicycle on the deferent orb, whose center is eccentric to the center of the World, fastness and slowness [should] occur with respect to the center of the World—as was said concerning the Sun. It therefore follows that the center of the epicycle should not cut equal arcs on the inclined [orb] in equal times. Yet such is not the case, since double elongation is not adjusted in the *zīj*es. An adjustment is not made because the center, even while moving along the circumference of the eccentric orb, cuts equal arcs on the inclined [orb] in equal time periods. If the mover of the eccentric [orb] were the inclined [orb], so that this motion would be uniform, the receding from and approaching the center of the World [by the epicycle center] would cease to exist. Therefore, one of two situations would be necessary: either [there would be] no approach of the center of the epicycle toward the center of the World nor receding from it, or else [there would be] different states in the motion of the center resulting in fastness and slowness; both of these [situations] are impermissible. This is a great doubt regarding this account that none of the practitioners of this discipline has raised any objection against, or, if anyone has, it has not reached us. There is a subtle solution for this doubt, but it is not appropriate to bring it up in this epitome. If at some other time, the blessed temper of the Prince of Iran—may God multiply his glory—would be pleased to command delving into this problem, an exposition will be made on it—God willing.

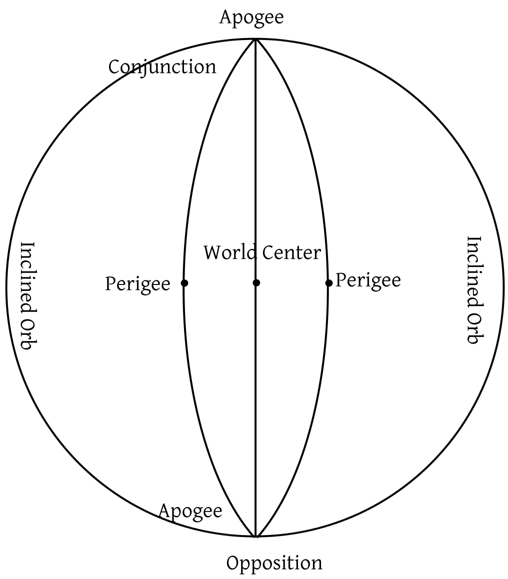
[25] This is the exposition of the configuration of the orbs of the Moon and their motions in longitude. There remains to explain the terms that are in common use among this group [of practitioners]. So, we say:

[26] **The lunar mean** is an arc on the parecliptic orb between the first of Aries and the [point of] intersection of the parecliptic with the latitude circle that passes through the endpoint of a line extending from the center of the World to the center of the epicycle and reaching the surface of the parecliptic. **The apogee of the Moon** is an arc on the parecliptic orb between the first of Aries and the [point of] intersection of the parecliptic with the latitude circle that passes through the end of a line extending from the center of the World to the apogee and reaching the surface of the parecliptic. **The center of the Moon and its double elongation** is an arc on the parecliptic orb between [the point] of intersection of the apogee's latitude circle with the parecliptic and the intersection of the epicycle center's latitude circle with the parecliptic. **The proper [anomaly] of the Moon** is an arc on the equator of the epicycle between the mean apex and the body of the Moon. **The adjusted proper [anomaly]** is an arc on the epicycle equator between the apparent apex and the body of the Moon. **The true position of the Moon** is an arc on the parecliptic orb between the first of Aries and the [point of] intersection of the parecliptic with the latitude circle that passes through the endpoint of a line extending from the center of the World to the center of the body of the Moon and reaching the surface of the parecliptic. **The node of the Moon** is an arc on the parecliptic equator between the first of Aries and the point of intersection of the inclined equator with the parecliptic equator. The illustration of the Moon's orbs is as follows:



[Figure 1]

[27] Practitioners of this science content themselves with the circles in black. From this illustration [Figure 2], the manner of the circuit of the center of epicycle in the inclined [orb,] and [the fact] that it reaches the apogee and perigee of the deferent orb twice will be understood:



[Figure 2]

[28] Other situations concerning the Moon, such as latitude, lunar eclipses, parallax, etc. will be treated later in their proper places—God willing.

CHAPTER SIX

An Exposition of the Orbs and Longitudinal Motions of the Upper Planets and Venus

[1] When the situation of the other planets is contemplated, [it will be seen that] Saturn, Jupiter, Mars, and Venus, all four, have direct and retrograde motion in common. The upper planets undergo combust at the midpoint of the direct motion period and are in opposition with the Sun at the midpoint of the retrogradation period; Venus undergoes combust in both situations. Venus's maximum distance from the Sun is not more than around 47 degrees. The direct motion of these planets is when they are distant from the Earth, and [their] retrogradation is when they are in proximity to the Earth. If one retrogradation is com-

pared with another retrogradation, and direct with direct, they will be found to be of varying periods. Whatever happens in a given part of the [zodiacal] orb in the shortest period will happen in its directly opposite part in the longest period, and that part of the [zodiacal] orb in which this is known to take place is displaced over a very long period of time, like the fixed stars. The circuits of these planets are not the circuit of the Sun but rather slant sometimes toward the north and sometimes toward the south.

[2] Therefore, through much contemplation, it became clear that if three solid orbs and three uniform motions are posited, [all of] these situations will fall into order in accordance with the mentioned principles. Each of these planets then has three orbs and three motions.

[3] As for the first orb, it is an orb whose center is the center of the World, its equator is in the plane of the zodiacal [orb], its convexity is contiguous with the concavity of the orb above it, and its concavity is contiguous with the convexity of the orb beneath it. As in Saturn, the convexity of this orb is contiguous with the concavity of the fixed stars, and its concavity is contiguous with the convexity of Jupiter's orb; and for Jupiter, its convexity is contiguous with Saturn's concavity, and its concavity is contiguous with Mars's convexity, and so on according to this pattern for Mars and Venus. This orb is called the parecliptic orb.

[4] As for the second orb, the orb is an eccentric, in the thickness of this [parecliptic] orb, as was said for the Sun, except that the plane of this orb's equator is inclined to the plane of the parecliptic equator. If a circle is conceived on the surface of the parecliptic orb in such a way that [this second orb's] equator be in the plane of that circle, that circle would intersect the parecliptic equator at two places, and that circle is called the inclined orb. Those two points [of intersection] are called the head and the tail, as was said concerning the Moon. And this orb is called the eccentric orb, as well as the deferent orb.

[5] As for the third orb, it is an epicycle orb, in the thickness of the deferent, as has been said for the Moon.

[6] As for the motions: the first motion is that of the parecliptic orb, which moves with the motion of the fixed stars. Since it is perceptible at the apogees and nodes, it is called the motion of the apogees. Since we have already said that the mover of this motion is the eighth orb, there is no need for positing this body [as the cause] for this motion, unless an eccentric orb is posited, [in which case] two complementary solids are certainly produced. The sum of these two complements with the eccentric is then an orb, since there is no empty place in the World, as is established in the principles of physics.

[7] The second motion is that of the deferent [orb]. It is 0;2 per day for Saturn; 0;5 for Jupiter; 0;31 for Mars; and 0;59 for Venus. Since this motion is perceptible in the epicycle center, it is called the motion of the center when the starting point of this motion is taken at the farthest distance; if the starting [point] of the motion is placed at the first of Aries, it is called mean motion.

[8] The third motion is that of the epicycle orb. It is 0;57 per day for Saturn, 0;54 for Jupiter, 0;28 for Mars, and 0;37 for Venus. Since this motion is perceptible in the bodies of the planets, it is called proper motion. The direction of these motions in these planets is opposite to what we said for the Moon, since for these planets it is in the sequence of the signs in the apical half but in the counter-sequence [of the zodiacal signs] in the perigean half. Due to this motion, the distance of the planets from the center of the World varies, as does faster and slower [motion] as well as direct and retrograde motion, because in the half where the motion conforms to the sequence of the zodiacal signs, the planet moves with direct motion, while in the other half where the motion is in the counter [sequence], the planet retrogrades. Retrogradation will be discussed in more detail later—God willing.

[9] Due to the motion of the epicycle around the deferent orb, a difference in ratio occurs between two retrogradations, or between two direct motions, or between two faster motions, or between two slower motions. This is because whenever the epicycle is at the apogee, a difference resulting in slower motion, which is due to the eccentricity of the deferent center—as was said for the Sun—is added to the difference that is due to the epicycle. And when the epicycle is at the perigee, that difference results in faster motion. Because the parecliptic moves with the motion of the fixed stars, the apogee and perigee, which are the midpoints of the periods of slower motion and faster motion [respectively], are displaced by the motion of the fixed stars. Inasmuch as the amount of proper motion of Saturn, Jupiter, and Mars is equal to the excess of solar mean motion over the motion of the centers of their epicycles, if we assume that the planet is at the apex and [hence] in combust, every day thereafter it will move away from the apogee by the amount of its proper motion, and this amount will be the distance between the centers of [their] epicycles and the Sun. Then, when each one of these two distances has gone half a revolution, the planet will have reached the epicyclic perigee and the Sun will reach opposition with the epicycle centers. When a full revolution has been made and the planet has reached the apex [once again], it will be in combust. The apex is the midpoint of the period of direct motion, and the perigee is the midpoint of the period of retrogradation. For this reason, these three planets always undergo combust when at the apex and at the midpoint of direct motion, and are in opposition with the Sun at the perigee and at the midpoint of retrogradation.

[10] Here is a question posed for the practitioners of this discipline. Why is it that Mars is closer to the Sun when it is in opposition to the Sun than it is when it is at combust? The answer is that since Mars's combust occurs at the apex, [the distance] between Mars and the Sun at the time of combust is the diameter of Mars's epicycle plus more,

due to the complement of its orb and the Sun's orb. And because its opposition to the Sun occurs at its epicycle's perigee, at that time [the distance] between it and the Sun is the diameter of the Sun's orb plus more, due to the complements. The diameter of Mars's epicycle orb is greater than the diameter of the Sun's orb; therefore, Mars is farther from the Sun during combust than during opposition.

[11] Since the motion of Venus's center conforms to the Sun's mean motion, the center of its epicycle is always aligned with the Sun; it therefore undergoes combust at the midpoint of its direct motion and the midpoint of retrogradation, i.e., at apex and perigee. Venus's distance from the Sun does not exceed the radius of its epicycle, which is around 47 degrees.

[12] Each of these planets has three anomalies: The first arises out of the two lines extending from the center of the World, [one] to the center of the epicycle and [the other] to the center of the planet's body. This is the independent, or second equation—as we mentioned for the Moon—except that [here] between apex and perigee it is additive and in the other half subtractive, contrary to the Moon, because the direction of the proper motions of these [planets] is opposite to that of the Moon's proper motion.

[13] The second anomaly comes about from the difference between [the amount of] the radius of these planets' epicycles at the farthest distance and the nearest distance. This is called the anomaly of the farthest and nearest distance. Its situation is close to what was described for the Moon, except in being additive or subtractive, since a difference occurs due to the way of calculation chosen by the author of the *Almagest*. This is because he calculated the second equation of the Moon assuming the epicycle at the farthest distance, for which reason the anomaly will be additive [as the second equation] increases, and subtractive as it decreases. He [also] calculated the second equation of

the planets at the mean distance, for which reason the anomaly is subtractive in the side of the farthest distance, when the second equation is additive, and when the second equation is subtractive, the anomaly is additive. In the side of the nearest distance the opposite occurs, as we mentioned for the Moon. This difference is due not to a variance in the configuration but to a variance in the [*Almagest*] author's way [of calculation].

[14] The third anomaly is that resulting from the alignment of the apex and perigee, for the diameter that passes through the apex and perigee in the epicycle orb is not aligned with the center of the deferent nor with the center of the World, but rather with a point whose distance from the center of the deferent, in the direction of the apogee, is equal to the distance of the deferent center from the center of the World. If one conceives of a line extending from that [abovementioned alignment] point that reaches the epicycle orb, and rotates with it, it is called the dirigent line. From this line, and a line extending from the center of the World to the center of the epicycle, an angle arises at the center of the epicycle, which is called the angle of the first equation. In the amount of this angle there occurs a difference between the mean apex, which is aligned with the [alignment] point and the visible apogee, which is aligned with the center of the World. Since the starting point of the proper [motion] has been taken from the mean apex, then this equation must be added to the proper [motion] in the half where the epicycle center is between the apogee and perigee, and subtracted in the other half in order to adjust the proper motion and make the visible apex its starting point. Even though [the direction of] the proper motion of the planets is opposite to that of the Moon, nonetheless because the alignment point in the Moon is on the side of the perigee, while in the planets it is on the side of the apogee, being additive or subtractive for this equation is as for the Moon.

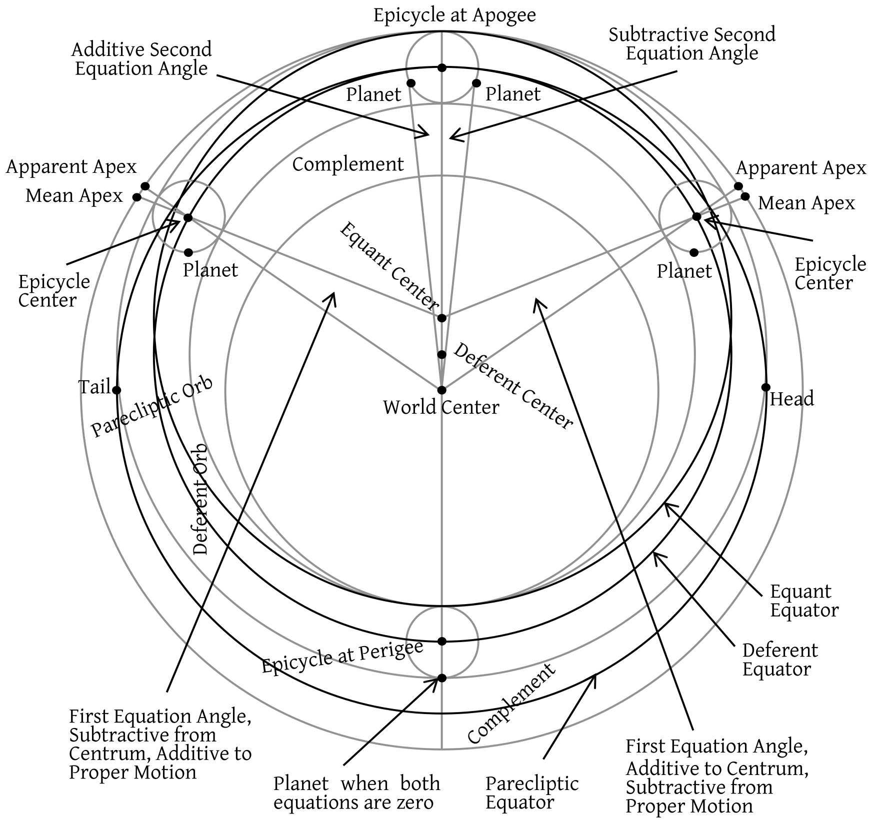
[15] The motion of the center of the epicycle for these planets is not uniform, neither with respect to the center of the World nor to the deferent center, but uniform with respect to the aforementioned [alignment] point. Then, a circle is conceived with the [same] size as the equator of the deferent with its center at the aforementioned [alignment] point; that circle is called the equant orb on which equal arcs will be cut in equal times. Thus, the difference of the motion of the center with respect to the center of the World is exactly brought about from the angle of the first equation—as was said in [the case of] the Sun. So in the half in which we add this equation to the proper [motion], it must be subtracted from the center; and in the half that it is subtracted from the proper [motion], it must be added to the center in order to adjust the center. This is not necessary for the Moon because the motion of the Moon's center is uniform around the center of the inclined orb.

[16] As for the doubt that occurred for the Moon, the exact same [doubt] applies to the motion of the epicyclic center on the equator of the deferent, with non-uniformity with respect to its center but with uniformity about another center eccentric to it. There follows one of the two aforementioned impossibilities: either there is uniformity about the deferent center so that the first equation becomes half of what it is in use; or else there is non-uniformity about the deferent center, so that the variations in the distance between the center of the epicycle and the center of the World become twice what it is known to be. The resolution of this doubt, about which none of the practitioners of this discipline has raised any objection and remains one of the mysteries of astronomy, will be explained at some other time—God willing.

[17] Practitioners of this discipline content themselves with circles, as was mentioned for the Moon. Now it is known that each of these planets has three solid orbs, two circles, and three simple uniform motions.

The orbs are: the parecliptic orb; the deferent orb; and the epicycle orb. The circles are: the inclined orb; and the equant orb. The motions are: the motion of the apogee; the motion of the center; the proper motion; and the fourth [motion], which is compounded of the motions of the apogee and the center and is the mean motion. If we consider the non-uniform, compound motions, the adjusted motion of the center, the adjusted proper motion, and the motion of the true position would be added.

[18] The distance between the eccentric center and the center of the World is: 3;25 for Saturn; 2;45 for Jupiter; 6;0 for Mars; and 1;2 for Venus—the radius of the eccentric being 60 degrees. Twice these amounts would be the distances between the equant center and the center of the World, which are used in the calculation of the first equation. The radius of the epicycle for Saturn is 6;30, for Jupiter 11;30, for Mars 39;30, and for Venus 43;10—the radius of the parecliptic being 60. These amounts are used in the calculation of the second equation. Between the head of a node and the apogee of Saturn is always 140 degrees; between the head and apogee of Jupiter 70 degrees; between the head and apogee of Mars 90 degrees; and between the head and apogee of Venus the same amount [90 degrees]. The terms used for them are close to what we have specified for the Moon, so we will not repeat them to avoid prolongation. The following is the illustration of the orbs of these planets:



[Figure 1]

[19] The black circles represent the ones that practitioners of this discipline posit. The latitudes of the planets will be explained later. This is the configuration of the orbs of these planets—God is all-knowing.

CHAPTER SEVEN

An Exposition of the Orbs and Longitudinal Motions of Mercury

[1] The various situations of Mercury are as have been stated for Venus, except that Mercury's elongation from the Sun is never more than about 27 degrees, and that Venus has a perigee opposite its apogee, while Mercury does not have a perigee opposite its apogee but

rather [another] apogee as well, though not with the same distance and slowness as the first apogee. [Mercury's] perigees are almost at the two trines with respect to the apogee. The speed of [Mercury's] apogee is the same as the speed of the fixed stars.

[2] Then, to regulate these divergences, Mercury needs four solid orbs, three circles, and four uniform motions. The first orb is an orb whose center is the center of the World, and whose equator is in the plane of the zodiacal equator, whose convexity is contiguous with Venus's concavity, and whose concavity is contiguous with the Moon's convexity. This orb is called the parecliptic.

[3] The second orb is an orb whose center is eccentric to the center of the World, and it is situated in the thickness of the parecliptic orb as is the case of the eccentric [orb] in the other planets, and is tangent to both surfaces of the parecliptic at two points, one being the apogee and the other perigee, as we have said. The plane of its equator is outside the plane of the parecliptic equator. This orb is called the dirigent orb.

[4] The third orb is another orb whose center is eccentric to both the parecliptic's center and the dirigent's center. It is located in the thickness of the dirigent orb, and also is tangent to both surfaces of the dirigent orb at two points directly opposite one another, one being the apogee and the other the perigee. Its equator is in the plane of the dirigent equator; the relation of this orb to the dirigent orb is like that of the eccentric orb to the parecliptic orb in the other planets. This orb is called the deferent orb.

[5] The fourth orb is the epicycle orb, embedded in the thickness of the deferent orb so that its surface is tangent to both of the [deferent's] surfaces at two points, as in the other planets. The planet Mercury is embedded therein as with the other planets.

[6] As for the motions, the first motion is the motion of the parecliptic with the speed of the fixed stars. Because this motion appears in the

apogee, it is called motion of apogee.

[7] Second is the motion of the dirigent orb in counter-sequence, like the mean motion of the Sun; and by this motion all the rest of the orbs are moved. This motion is perceived in the apogee and perigee of the eccentric deferent [orb], and this motion is called the motion of the deferent apogee. Since through this motion the center of the deferent has a circuit around the dirigent center, this circuit is called the deferent orb of the deferent center, and this motion is called by the same name.

[8] The third motion is the motion of the deferent orb in the sequence of the zodiacal signs, equal to twice the Sun's mean speed. Since this motion is perceived in the epicycle center, this motion is called the motion of the center.

[9] The fourth motion is the motion of the epicycle orb in the same direction as the orbs of the epicycles of the other planets; it is 3;6 daily and this is called the proper motion.

[10] If a circle is conceived on the [exterior] surface of the parecliptic orb in whose plane the deferent equator lies, that circle is called the inclined orb. This [circle] necessarily intersects the parecliptic equator in two places that are called the head and the tail, and the motion of the fixed stars can be perceived in them.

[11] Then, when the center of the epicycle is at the deferent apogee, the deferent apogee will be coincident with the apogee point of the dirigent, and both apogees will be coincident at one common point on the upper surface of the parecliptic orb; the center of the epicycle will be at the farthest distance from the center of the World. Thereafter the deferent apogee moves in counter-sequence by the amount of the mean motion of the Sun, and the center of the epicycle moves in sequence by double the amount of the Sun's mean. By this [same] amount is the center elongated from the deferent apogee, and it is

elongated from the dirigent apogee by the amount of the Sun's mean. This is the amount of the motion of Mercury's center that is used in the *zīj*es. When the starting point is assumed to be the beginning of Aries, [the motion] is compounded of the motion of the dirigent apogee and this above-mentioned motion, whence it is called the mean motion [of Mercury].

[12] Thus, the dirigent apogee is always between the deferent apogee and the epicycle center, such that when each has traversed a quarter [revolution] of the dirigent orb, and both reach quadrature with respect to the dirigent apogee, the epicycle center will have reached the perigee of the deferent orb, which is directly opposite the apogee. When each has traversed another quarter [revolution] of the [dirigent] orb, the deferent apogee and the epicycle center will again meet opposite the dirigent apogee. Thus, the deferent apogee and dirigent perigee will meet, and the epicycle orb will be at the farthest distance from the center of the World—other than its first farthest distance. Then they move apart again, the deferent apogee going into the first half, and the epicycle center into the second half. At quadrature with respect to the dirigent apogee they are opposite once again, and the epicycle center reaches the deferent perigee. From there they are separated, and the two meet at the dirigent apogee. Since the second apogee—which is opposite the dirigent apogee—is nearer to the center of the World than the first apogee—which is compounded of both apogees—there is no true perigee—i.e., maximum proximity to the center of the World—at the two quadratures with respect to the dirigent apogee, as was said for the Moon. Rather, the true perigee in both directions is where it is compounded of both perigees [of the dirigent and deferent orbs], and in any case it is nearer to the opposition [point] of the dirigent apogee. Thus, the two perigees are almost at the trines with respect to the dirigent apogee, and at the sextiles with respect to the [point] opposite [the apogee].

[13] Because of the epicycle orb and its motion, retrograde and direct motions are produced; and because of the eccentric and its motion, the difference between one retrogradation and [another] retrogradation, and between one direct motion and another direct motion, are produced. On account of the dirigent orb and its motion, the epicycle center reaches the apogee twice per revolution, provided that one apogee is higher than the other, and [the epicycle center also reaches] the perigee twice per revolution provided that two perigees are equal; for if the motion were not in the counter-sequence [of the zodiacal signs], the apogee would not meet the center in its circuit, as we have said for the Moon. If this orb were not eccentric, both apogees would be equal in distance, as it is for the Moon. Because of the parecliptic orb, the dirigent apogee moves in sequence with the speed of the fixed stars. By these orbs and motions the above-mentioned anomalies are ordered.

[14] Mercury also has three anomalies. First is the anomaly from the radius of the epicycle, i.e., the second equation, which is [the angle between] two lines extending from the center of the World to the center of the epicycle and to the body of the planet. Its maximum is in the amount of the radius of the epicycle, and this amount is 22;30.

[15] Second is the anomaly due to the epicycle being at different distances, i.e., the inequality of farthest and nearest distances.

[16] Third is the anomaly due to the alignment of the epicycle's diameter with a point that is other than the above-mentioned center points. This is the first equation.

[17] The state of being additive or subtractive in these anomalies is just the same as being additive or subtractive in the other planets.

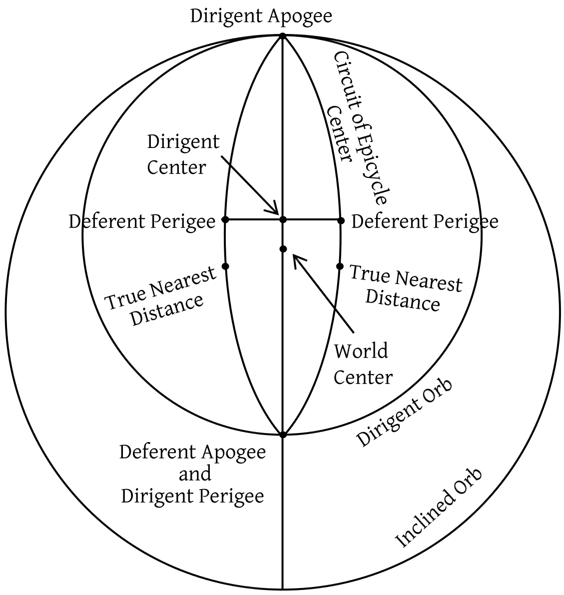
[18] The point with which the mean apex and perigee are aligned for this planet lies between the dirigent center and the center of the World. Thus, when the deferent apogee and dirigent apogee coincide,

these four centers are along one line: first is the center of the World; above it is the above-mentioned point which is the center of the equant circle; above it is the dirigent center; and above it is the deferent center. The distance between each two of these centers is 3;0, assuming the radius of the eccentric to be 60. The equant circle for this planet is, as in the other planets, the same size as the deferent equator; its position, however, is toward perigee, unlike the other planets. Since the distance of the deferent center from the dirigent center is equal to the distance of the equant center from it, both centers are thus on the circumference of a small circle [known as] the deferent of the deferent center. Since the deferent center moves around this circle, it reaches the equant center once in every revolution, [whereupon] the equant and deferent coincide, and afterwards are separated once again. They coincide when the epicycle center is at the closer apogee. Practitioners of this discipline are content to set forth circles and put the deferent orb of the deferent center in place of the dirigent orb. The other circles are in the manner that was mentioned previously.

[19] Therefore, from this discussion [the following] has been established: four solid orbs, namely, the parecliptic orb, the dirigent orb, the deferent orb, and the epicycle orb; three circles, namely, the inclined orb, the equant orb, and the deferent orb of the deferent center; four simple uniform motions, namely, the motion of the apogee, the motion of the center, the motion of the dirigent, and the proper motion; two uniform compound motions, namely, the mean motion and the motion of the center used [in the *zīj*es];[1] and three compound motions, namely, the adjusted motion of the center, the adjusted proper motion, and the motion of the true position. The meanings of the names are the same as has been previously stated.

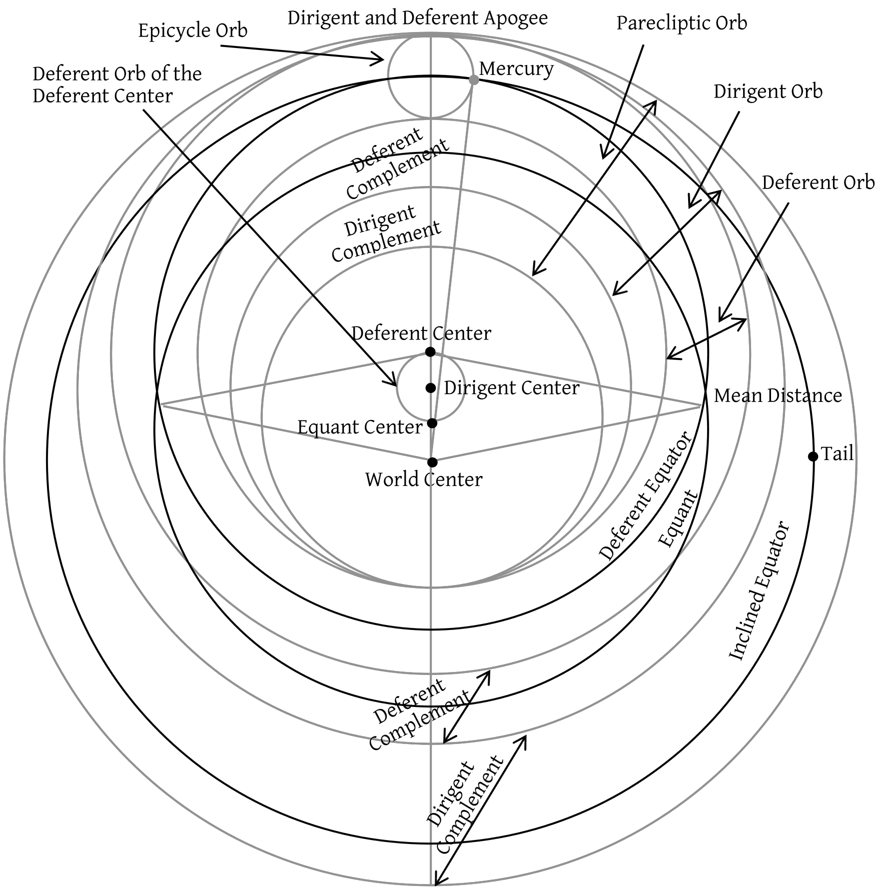
1. For "the motion of the center used [in the *zīj*es]," see paragraph 11 of this chapter.

[20] Between the head and the apogee of Mercury is 270 degrees. Since this planet has two eccentric orbs, there are four complementary solids: two from the dirigent orb and two from the parecliptic orb. Here is an illustration of the circuit of the center of the epicycle in relation to the centers of the World:



[Figure 1]

[21] The doubt of which we spoke concerning the equant and the eccentric applies to these orbs as well, and the solution also is as in the other [planets]. The illustration of the orbs of Mercury is this:



[Figure 2]

[22] Those parts that have been illustrated in black are the parts practitioners of this discipline are satisfied with. This much is sufficient for this chapter—God is all-knowing.

CHAPTER EIGHT
On the Latitudes of the Six Planets

[1] Turning to the latitudes of the planets: from what we have previously stated, namely that the center of the body of the Sun is always on the equator of the eccentric orb and that the equator of the eccentric orb is in the plane of the parecliptic equator whose plane is part of

the plane of the zodiacal orb, it is obvious from this that the Sun is always in the plane of the zodiacal orb and thus has no latitude. For this reason, the equator of the zodiacal orb is also called the solar circuit.

[2] As for the Moon, we have stated that it is on the equator of the epicycle orb, and the equator of the epicycle orb is in the plane of the eccentric orb's equator; the eccentric equator is in the plane of the equator of the inclined orb. Therefore, the Moon is always in the plane of the equator of the inclined orb, and the inclined equator is its circuit. Because the inclined equator intersects the parecliptic equator at two places, [i.e.,] the head and the tail, therefore when the Moon reaches the point of intersection it will be on the parecliptic equator and will have no latitude. In all other positions it will have latitude. While it is between the head and the tail, its latitude is northerly because the head is the northern crossing point; in the other half, it is southerly because the tail is the southern crossing point.

[3] When the point of intersection, i.e., the head, is taken as the starting point of the motion of the Moon's true position, which is a motion combined of the sum of the true motion of the Moon and the motion of the head, the latitude occurs due to this motion, and it is called the argument of latitude. When this argument of latitude is less than half a revolution, the latitude is northerly; when it is more, southerly. When a circle is imagined that passes through the four poles of the inclined and parecliptic [orbs], each half of the inclined orb is divided into two halves as well. Then, the first quarter, which the Moon enters as it passes the head, is the ascending latitude in the north and additive; in the second quarter, it is descending and subtractive; in the third quarter, which is after it passes the tail, it is descending in the south and additive; and in the fourth quarter, it is ascending and subtractive. This ascension and descension has been so established because the north[ern side of the zodiacal orb], from the viewpoint of the people of northern habitations, is of greater elevation than the south[ern

side]. The maximum latitude of the Moon in both directions is equal to the maximum declination of the inclined [orb] from the parecliptic, which is 5 degrees. Other than this, the Moon has no latitude.

[4] As for the upper planets, i.e., Saturn, Jupiter and Mars, each one has two latitudes. One is due to the declination of the inclined orb from the parecliptic orb, as has been said about the Moon. Since the center of the epicycle is on the equator of the eccentric, and the equator of the eccentric is in the plane of the inclined orb, this latitude is the latitude of the center of the epicycle orb. When the center is at either one of the two points of intersection of the parecliptic and inclined [equators], the latitude is zero. In other positions there is a latitude. The maximum latitude in both directions is: 2;30 for Saturn; 1;30 for Jupiter; and 1;0 for Mars. The circumstances of this latitude are exactly as has been said for the Moon.

[5] The second latitude is due to the apex and perigee of the epicycle orb, because the apex and perigee for these planets, or rather the plane of the equator of their epicycle orbs, does not remain fixed in the plane of the inclined equator, but rather the diameter that passes through the apex and perigee intersects the plane of the inclined orb. The inclination of the apex is always toward [one] side of the zodiacal orb, and the inclination of the perigee is toward the other side. Every time the epicycle center reaches one of two points, either the head or tail of that planet, this declination is nullified, and the plane of the epicycle equator coincides with the plane of the inclined orb. As long as the center of the epicycle is in the northern side, the inclination of the apogee is toward the south and the inclination of the perigee is toward the north. The maximum of each one [occurs] when the first latitude is at its maximum and [its] decrease [occurs] with the decrease [of the first latitude] simultaneously. When the center of the epicycle is toward the south, the inclination of the apex is toward the north and the inclination of the perigee is toward the south. The maximum

is like the maximum of the first latitude and is additive or subtractive to that latitude.

[6] The maximum of this latitude for each of the apex and the perigee, when the center of the epicycle is in the north, is: Saturn, 0;26, 0;32; Jupiter, 0;23, 0;34; Mars, 0;52, 3;21. In the south, they are: Saturn, 0;28, 0;35; Jupiter, 0;25, 0;38; Mars, 0;56, 6;07. When the [first and second] latitudes are combined, the inclination of Saturn's apex and perigee in the north is 2;4, 3;2 and in the south 2;2, 3;5; the inclination of Jupiter's apex and perigee in the north is 1;6, 2;4 and in the south 1;5, 2;8; the inclination of Mars's apex and perigee is 0;8, 4;21 in the north and 0;4, 7;7 in the south.

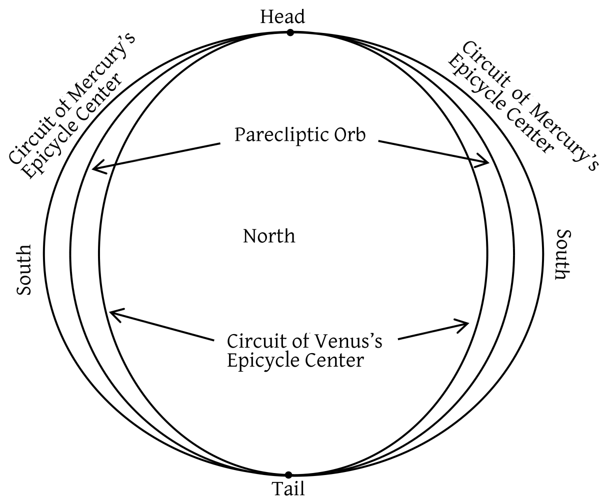
[7] In these planets, the diameter of the epicycle orb that passes through the two mean distances is always in the plane of the inclined orb. Since this latitude is due to a motion that is in the plane of the epicycle equator, it must assuredly have a mover. Abū ʿAlī ibn al-Haytham, who was one of the great scholars in the mathematical sciences, wrote a treatise on this matter and transformed each one of the epicycle orbs of these planets into three orbs enclosing one another: one is moved by the proper motion; another moves its equator in such a way that the diameter of the apex and perigee are inclined to the north and south; and the third is an orb that moves those two orbs in the counter-sequence so that the disruption caused by the motion of the second orb in the position of the first orb is eliminated.

[8] Yet even with this determination, this anomaly is not regulated, and in addition several other distortions come into being; but this is not the place to explain them. The author of the *Muntahá al-idrāk* says that the endpoint of the diameter that passes through the apex and perigee moves on the circumference of a small eccentric circle whose relation to another small circle is exactly that of the planet's eccentric to its inclined [orb]. But from what he says no defect that must be

removed is eliminated. Abū ʿAlī Haytham's treatise is still much nearer to the truth than this, and these words have been quoted exactly from Abū ʿAlī's and others' books.

[9] As for Venus and Mercury, both planets have three latitudes each. The first is the latitude from the inclination of the inclined orb from the parecliptic. This latitude is always northerly for Venus and southerly for Mercury. Practitioners of this discipline have said that the plane of the inclined orb intersects the plane of the parecliptic orb, as we have said; however, its inclination from that plane is not fixed. Rather, when the maximum has been reached, the plane of the inclined turns around and heads toward the plane of the parecliptic until both planes coincide. Then the inclined [orb] crosses the parecliptic [orb], and the half that was northerly becomes southerly and the half that was southerly becomes northerly, and so on until it reaches the maximum [inclination]; then it turns around, and so on and so forth. Then, when the center of the epicycle reaches the point of the head, the inclined plane will then coincide with the parecliptic plane. When the coincidence ceases to be and there is an inclination, the epicycle center of Venus is northerly and the epicycle center of Mercury is southerly. When each of these two centers reaches its maximum inclination, i.e., the midpoint between the two nodes, the inclination between the two planes attains its maximum. Thereafter the epicycle center heads toward the tail and the inclined plane reverts until, when the epicycle center reaches the tail, both planes coincide. Then, when one plane moves away from the other, the half that was northerly at that moment becomes southerly, and the half that was southerly becomes northerly. For Venus, the shift will have come in the half that was originally southerly but is now northerly, while for Mercury the shift will have come in the half that was originally northerly but is now southerly. Thus, Venus's epicycle center is always to the north of the parecliptic except at the two points of intersection at which it is

then on the parecliptic, and Mercury's epicycle center is always south of the parecliptic except at the two points of intersection at which it is then on the parecliptic. Since the eccentric is in the inclined plane, the apogee is northerly for half a revolution and southerly for half a revolution. This is an illustration of the circuit of the epicycle center for these two planets in relation to the parecliptic:



[Figure 1]

[10] The maximum latitude of Venus on either side [of the zodiacal equator] is $\frac{1}{6}$ of a degree; the maximum latitude of Mercury $\frac{1}{2}+\frac{1}{4}$ [of a degree]. Another orb must be posited for this motion, which the ancients did not do. This has been also indicated by Abū ʿAlī al-Haytham in the aforementioned treatise, but this is not the place to discuss that tract [of Ibn al-Haytham].

[11] The second latitude is the latitude of the diameter passing through the apex and perigee. When the center of the planet is at the midpoint of the two nodes, i.e., the head and tail, this latitude is zero on both sides [of the zodiacal equator], and the diameter that passes through the apex and perigee is in the plane of the inclined orb. When [the center of the planet] is at the head or tail, the inclination of the

diameter is at its maximum. An explanation of this is that when the epicycle center is at the midpoint between the head and the tail, which position is Venus's apogee and [the point] opposite Mercury's apogee, the apex begins to incline toward the north and the perigee begins inclining toward the south. Then, when it reaches the point of the tail, the apex is at its maximum northerly inclination and the perigee is at its maximum southerly inclination. When it reaches the midpoint between the tail and the head, the inclination disappears. After that, the apex begins its inclination toward the south and the perigee toward the north until, when it reaches the head, it will have attained its maximum. The maximum inclination for Venus's apex toward the north and south is 1;2, and the maximum inclination of its perigee in both directions is 6;20. The maximum inclination of Mercury's apex in both directions is 1;45, and the maximum inclination of its perigee in both directions is 4;4.

[12] As for the third latitude, which is called the slant, winding, or slope, this is the inclination of the diameter that passes through the two mean distances. The one-half of this diameter that is eastern and at which the planet appears before the Sun rises, is called the matutinal distance. The other half that is western and at which the planet appears after the Sun sets is called the vespertine distance. This inclination reaches its maximum when the epicycle center passes away from the head and is between the head and tail or between the tail and head, like the first latitude. When the center is at one of the two points of intersection, this inclination is zero. Then, when the epicycle center passes away from the head, the eastern side heads toward the north, and the western side toward the south, until when the center reaches the midpoint of the two nodes—i.e., the apogee for Venus and the [point] opposite the apogee for Mercury—this latitude will have reached its maximum. From there it keeps decreasing until it reaches the tail where the diameter is in the inclined plane. After passing away

from the tail, the eastern side heads southward and the western side northward, until it reaches its maximum at the midpoint, which is [the point] opposite Venus's apogee and Mercury's apogee, and then once again begins to decrease. The maximum of this latitude for Venus in both directions is 2;30, and for Mercury it is 2;15 in the apogean half and 2;45 in the perigean half. This latitude is the reciprocal of the second latitude, i.e., when that latitude is at its maximum, this latitude is zero, and when this latitude is zero, this latitude reaches its maximum.

[13] Abū ʿAlī Haytham has posited for each of Venus and Mercury five epicycle orbs, enclosing one another: one for the proper motion; the second for the inclination of the diameter of the apex and perigee; the third for maintaining the position that would otherwise be displaced due to [the second orb]; the fourth for the slant; and the fifth for maintaining the position that would otherwise be displaced due to the [fourth orb's] motion. This is [our] words concerning the latitudes of the six planets—God is all-knowing of the Truth.

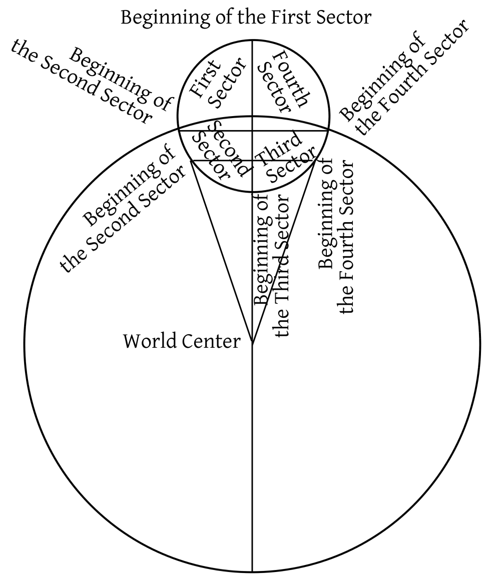
CHAPTER NINE
An Exposition of the Planetary Sectors

[1] Each of the eccentric orbs and orbs of epicycles mentioned previously can be divided into four parts, each part of which is called a sector. Practitioners of this science disagree over where to place the beginning of the second and fourth sectors, but there is no dispute over the beginning points of the first and third sectors, which are the apogee and perigee or the apex and perigee.

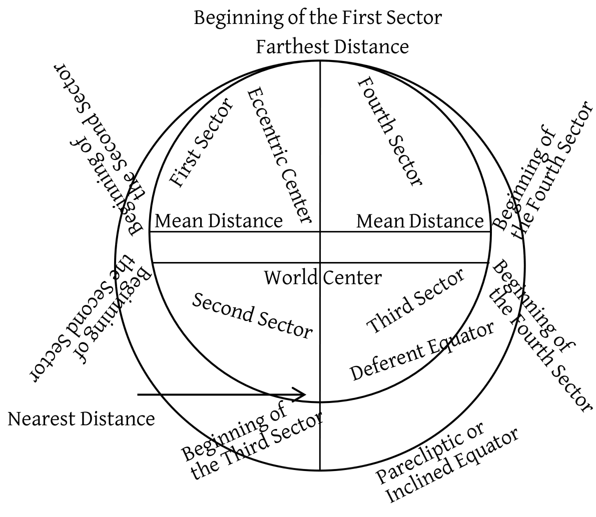
[2] One group says that the initial points of the second and fourth sectors are the two points of mean distance, just as the beginning points of the first and third sectors are the points of the farthest and nearest [distances]. This being so, then a diameter must be assumed in the eccentric orb that passes through the apogee and perigee, and a line

must be assumed that both passes through the midpoint between the two centers and intersects that diameter at right angles, thereby dividing the orb into four parts. In the epicycle orb, a diameter must be assumed that passes through the apex and perigee, and [another] line [must be assumed] that passes through the two points of intersection of the deferent equator with the equator of the epicycle in order for the sectors to be determined according to this group's view.

[3] Another group has said that since the eccentric orb and the epicycle orb have been distinguished by an anomaly called the equation, and the first and third sectors begin where the equation is zero, so the second and fourth sectors [should] begin where the equation reaches its maximum. This being so, then a line that is assumed in the eccentric orb must pass through the center of the inclined [orb] and be at a right angle to the diameter. And in the epicycle, the line [that is assumed] must pass through the two points of tangency on both sides [of the epicycle] made by two lines extending from the center of the World tangent with the epicycle orb, since the maximum of the equation is at these positions. These cases can be seen in the two [following] illustrations that have been drawn. Therefore, when the planets are in the first and second sectors, they are descending, and ascending in the third and fourth sectors. In the fourth and first sectors, they are in the upper half, and in the second and third sectors they are in the lower half—God is all-knowing.



[Figure 1]



[Figure 2]

CHAPTER TEN

An Exposition of Retrogradation and Direct Motion, and Orientality and Occidentality

[1] Ptolemy stated in the *Almagest* that retrogradation of planets can be [explained] either by epicycle orbs or by eccentric orbs. Neither one makes any difference in the positions of these two orbs, on the condition that the ratio of the radius of the parecliptic orb to the radius of the epicycle be the same as [the ratio of the radius of the parecliptic orb to] the distance between the two centers on the assumption of the eccentric [orb]. And this [assumption] can be represented when the motion of the eccentric is assumed to be in the opposite direction to the motion of the parecliptic. Inasmuch as practitioners of this discipline have established the eccentric for another purpose, they have set the epicycle for [explaining] retrograde and direct motion.

[2] Now, if the motion of the epicycle center on the circumference of the deferent is more than the motion of the planet on the epicycle, the planet does not retrograde, but rather is faster in the half whose two motions are in the same direction, with a motion that is a combination of the two motions. In the other half, whose motion of the epicycle is counter to the motion of the center, it is slower by the amount of the excess of the motion of the center over the proper motion. This conception is what has been related for the Moon. If the proper motion with respect to the center of the World is additive to the motion of the center, as it is for the five planets, it follows that retrogradation occurs with respect to the center of the World in the half where the motion of the epicycle is counter to the motion of the center.

[3] In the *Almagest*, it has been proven that when a line goes from the center of the World to the epicycle orb and [then] passes through it, if half of the part [of the line] falling inside the epicycle to that falling

outside between [the epicycle] and the center of the World has the same ratio as that of the motion of the center to the planet's proper motion, then when it reaches that line in the opposite half, [the planet] will become stationary. In an area where the ratio of half the inside part to the outside part is less than the ratio of the motion of the center to the proper motion, the planet moves directly, whereas in the other area, where it is more, it retrogrades. So, there will be no retrogradation in any epicycle that has a ratio of its radius to the line between it and the center of the World less than the ratio of the motion of the center to the proper motion. In any epicycle where these two ratios are equal there will be a station but no retrogradation.

[4] Now that these preliminaries are understood, let us say that when these five planets move in the sequence of the [zodiacal] signs in the upper half of their epicycles, their speed at that time with respect to the center of the World is faster than the mean. When they pass that [half] reaching the mean distance of the epicycle orb, the [planets] descend on a straight line with respect to the center of the World; hence the motion of the epicycle is not perceptible, and the planet's motion is its mean motion. Thereafter, they move in the counter-sequence [of the zodiacal signs]; as long as their motion in the epicycle is less with respect to the center of the World than the motion of the center of the epicycle, their speed is slower but is still direct. When the two speeds are equal, the planet is stationary. When the speed of the planet on the epicycle overtakes the speed of the center, [the planet] retrogrades. When it reaches perigee, it will be at the midpoint of retrogradation. On the other side, as before, it will become stationary, then slow down, then be at mean speed, then speed up, and then at apex it will be in the middle of direct motion. Were it not for the motion of the center, the planet would retrograde for approximately a half, such that it would move in the counter-sequence [of the zodiacal signs] with respect to the center of the World. This is the situation of

retrogradation and direct motion.

[5] Since all the planets combust at the apex, the Sun overtakes the upper planets after combust; therefore, they rise before the Sun does. They are then called oriental [to the Sun] until there are 60 degrees between them and the Sun. When they reach near the Sun's trine, they become stationary. Then they retrograde, and, at the midpoint of retrogradation, which occurs at perigee, they will be in opposition to the Sun. After that, near the second trine, they become stationary, and then they move directly. When there is less than 60 degrees between them and the Sun, they set after the Sun and are occidental [to the Sun] until they reach the Sun at apex.

[6] When Venus and Mercury pass the apex, they precede the Sun and therefore set after it does. They are then called occidental [to the Sun] until they reach *ribāṭ-i aʿẓam*, which is the maximum distance, and there their speed is slower. Thereafter they retrograde, and at the midpoint of retrogradation they reach the Sun. This is the second combust, which occurs at perigee. When they pass beyond, they will rise before the Sun and become oriental [to the Sun] and again move directly. When they reach the maximum distance, their speed increases until the initial state of affairs is reached. In one half of their cycle they are occidental [to the Sun], and in the other half they are oriental [to the Sun], [the halves being] opposite to those of the upper planets—God is all-knowing of the Truth.

CHAPTER ELEVEN
An Exposition of Parallax of the Lower Planets

[1] Since the positions of the planets on the zodiacal orb are determined by the line that passes from the center of the World through the center of the body of the planet and reaches the surface of the sphere of the zodiacal orb, the line that reaches the planet and the

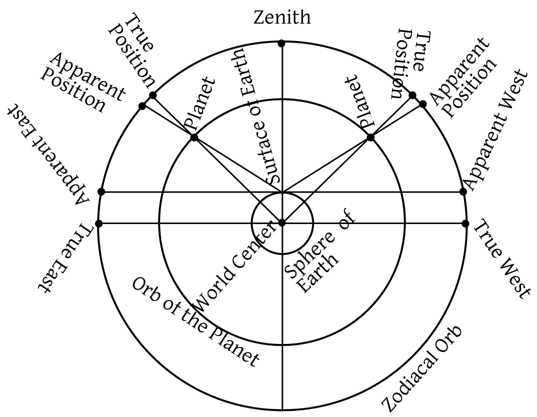
surface of the zodiacal orb from the viewpoint of an observer on the surface of the Earth will no doubt differ from the former line. The difference between these two lines is due to the radius of the Earth, and the closer a planet is to the Earth the greater this difference appears. Thus, a planet's position with respect to the center of the World is the planet's true position, while the position of the planet with respect to the surface of the Earth is its apparent position. In all cases the apparent position is closer to the horizon than the true position, because the line extending from the surface [of the Earth] falls below the line [extending] from the center as it passes through the planet. In all cases, if a plane that divides the orb into two halves, one visible and the other invisible, is tangent to the visible surface of the Earth, the visible half will be smaller than the invisible half. This is because this plane is parallel to the plane that divides the orb into two [equal] halves and passes through the center with respect to the Earth's radius. This difference [i.e., parallax] is perceptible in the orbs up to the orb of Mars, but in Mars's orb it is not perceptible because in relation to Mars's circuit the Earth is like a point, and the visible half is equal to the invisible half. This difference is greatest in the Moon, which is the nearest of all the [celestial] bodies [to the Earth].

[2] Parallax on an altitude circle is then an arc on the altitude circle between the endpoint of the line of center and the endpoint of the line of sight on the surface of the zodiacal orb.

[3] In loci where the equator of the zodiacal orb passes through the zenith, and the planet is on the [zodiacal] equator, and the [zodiacal] equator coincides with the altitude circle, the parallax in the altitude circle is longitudinal parallax only, with no latitudinal parallax. When [both] the pole of the zodiacal orb and the planet are on the meridian, the meridian circle is [both] the altitude circle and the latitude circle; in this case, the planet has parallax in latitude but no parallax in longitude. Likewise, when the planet's altitude circle passes through the

two poles of the zodiacal orb, which happens when the planet is at the midpoint between ascendant and descendant. If the planet is at mid-heaven at the zenith point, then there will be no parallax, neither in longitude nor in latitude. In all other positions, the parallax will be a combination of the longitude and latitude [components].

[4] The maximum parallax of the Moon is about $1+\frac{1}{2}+\frac{1}{4}$ ° when it is at the nearest distance, and 54 when it is at the farthest distance. During a lunar eclipse, it is never more than 1°;4′. The maximum solar parallax is 3′ when the Sun is at the nearest distance and bordering on one minute when it is at the farthest distance. Here follows an illustration of parallax—and God is all-knowing:



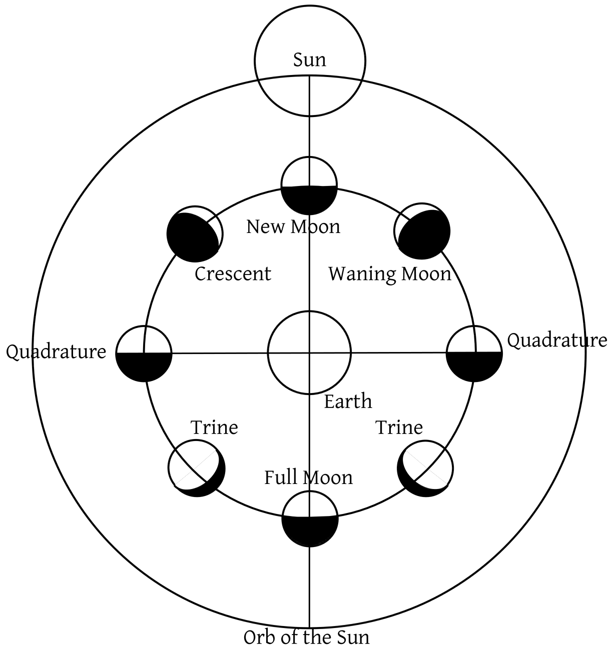
[Figure 1]

CHAPTER TWELVE
On the Reason for the Increase and Decrease in the Moon's Light

[1] The body of the Moon is a spherical body that is thick and smooth but not, in actuality, luminous. Every body that is thick and smooth, when facing a luminous body, will be illuminated by the rays [of that luminous body], and, like mirrors, water, etc., reflect those rays onto things facing it. Likewise, the Moon is illuminated by facing the Sun

and reflects its rays. One half [of the Moon] always faces the Sun, and therefore one half of it is illuminated while the other half is dark and of its actual color.

[2] At conjunction, the half that faces the Sun is upward, and the half that faces us is in its actual color and dark; therefore, the Moon is said to be obliterated. As it departs from conjunction, the part of the illuminated half that faces us and is in the shape of a crescent is enclosed by two semicircles: one semicircle dividing illuminated from dark; and the other semicircle dividing the visible from the invisible. The farther the Moon gets from the Sun, the greater the crescent shape becomes, until it reaches quadrature; [then] one half of the Moon will be visible, and the semicircle dividing illuminated from dark appears as a straight line dividing the Moon into two halves. Furthermore, when it reaches opposition, the illuminated half, which faces the Sun, will face us exactly the same, and the Moon will then be full. After it departs from opposition, contrary to the initial state, the Moon begins to become dark, and [its dark part] grows larger until the second quadrature, when [the dark part] is half. After this, when it reaches the "obliterated state" [at conjunction], it has returned to the initial state. Here follows an illustration of the Moon's positions with respect to the Sun:

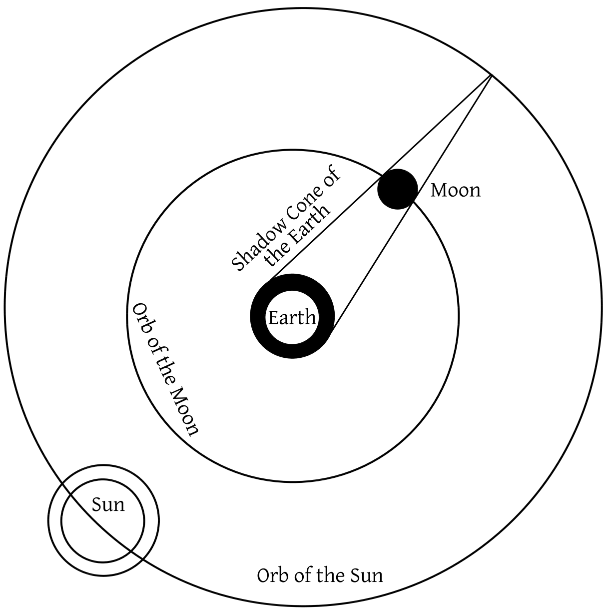


[Figure 1]

CHAPTER THIRTEEN
On the Reason for Lunar and Solar Eclipses, and the Interval Between Two Lunar or Two Solar Eclipses

[1] **Lunar Eclipse.** Since the light of the Moon is from the Sun, whenever the Earth interposes between the Moon and the Sun, it blocks the Sun's light from the [Moon], so that it appears in its actual color. This situation is called a lunar eclipse. It is of course conditional in this case that the Sun, Moon, and Earth, all three, be aligned with one another. Since the Sun is always on the zodiacal equator and the Earth is at the place of the equator's center, inasmuch as the Earth's center is the center of the zodiacal equator, then whenever the Moon is in opposition and does not have much latitude, it will fall into alignment with the Sun and the Earth, and then the lunar eclipse will occur. But if [the

Moon] has a [notable] latitude, it will diverge from this alignment, and there will not be a lunar eclipse. When Sun[light] falls on the Earth, the Earth casts a shadow in the direction opposite its alignment with the Sun. If the Moon's latitude equals the radius of the shadow circle, which [can] touch the Moon, plus the radius of the Moon, the Moon will touch the shadow but no eclipse will occur. If it is greater, the Moon will not even touch [the shadow circle]. If it is less, but greater than the excess of the shadow's radius over the radius of the Moon, part of the Moon will be eclipsed. If it is equal to that excess, the Moon will be fully eclipsed, but there will be no duration. If it is less, there will be a duration. If opposition does not occur at night, the eclipse is imperceptible. Parallax has no effect on a lunar eclipse, because as the Moon falls into darkness, its visibility is the same for all localities. Since it is the Moon that reaches the shadow by its own motion and then passes away from it, a lunar eclipse always begins from the eastern side, and reappearance also begins from that direction—God is all-knowing. And this is the figure of a lunar eclipse:



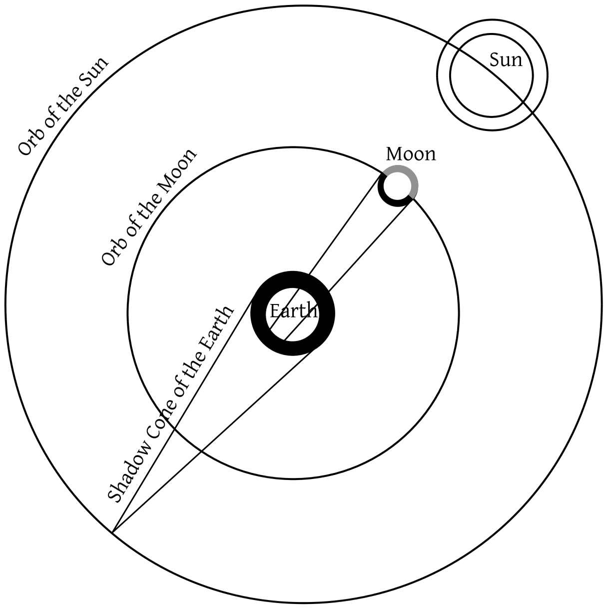
[Figure 1]

[2] **Solar eclipse.** When there is a conjunction such that the body of the Moon interposes between the eyes of individuals and the body of the Sun, the [body of the Moon] blocks the sunlight from them. The side of the Moon's body that is toward them appears black and of its actual color, so it appears that the Sun is blackened; this state of affairs is a solar eclipse.

[3] Since the Sun is on the [zodiacal] equator, for a solar eclipse to occur, the Moon must also be near the [zodiacal] equator; this occurs when its latitude is small. In this instance, parallax has a great effect, inasmuch as sometimes there may be a true conjunction and the Moon has no latitude but still no solar eclipse occurs because the Moon [apparently] has deviated [from conjunction]. When there is an apparent conjunction, a solar eclipse certainly occurs. For the reason we have stated in [the section on] parallax, true conjunction is always closer to the meridian than apparent conjunction. It may also happen that a solar eclipse will occur in one place but not in another.

[4] The apparent latitude is that which is adjusted by parallax. Thus, when there is no apparent latitude, the center of the Moon will be aligned with the center of the Sun [and] the entire body of the Sun will be eclipsed. However, there will be no duration to the solar eclipse, since the circle of the Moon's surface is not larger than the circle of the Sun's surface. When the apparent latitude is less than [the sum of] the radii of both bodies, part of the Sun is eclipsed. If it is equal to that [sum], the Moon appears to be tangent to the Sun, and no eclipse will occur. If conjunction occurs at night, the eclipse will not be visible. And since it is the body of the Moon that passes across the Sun, the beginning of the solar eclipse and its reappearance are always from the western side. Because a northerly latitude appears less in northern localities due to parallax, and a southerly latitude appears greater, there [may] be a solar eclipse with a greater northerly latitude but not with a southerly [latitude, with the same amount]; and the opposite [is the

case] in southern localities. This is an illustration of the solar eclipse:



[Figure 2]

[5] It should be known that usually between any two [consecutive] lunar or two [consecutive] solar eclipses, there will be a period of six lunar months, because when the Moon and Sun come together at one node [in conjunction or when they are at the two nodes][1] in opposition, a solar or lunar eclipse occurs. Thereafter an eclipse will not reoccur until the Sun reaches the other node. The node condition is due to the fact that the Moon does not have much latitude [at the nodes]. Sometimes a solar eclipse may occur, for example, at a distance from the head, whereby the Sun has passed beyond the head several degrees, and again several degrees before the Sun reaches the tail another solar eclipse may occur, and five months elapse between the two [consecutive solar eclipses]. In lunar eclipses too this is possible. In solar eclipses, however, if the first time it has passed the tail and the

1. The additional bracketed phrase is found in MSS M and F.

second time it has not reached the head, no solar eclipse will occur within five months, because in both [cases] the latitude was southerly. For southerly latitudes not many solar eclipses are possible. [However] this is possible in the [case of the] Moon, because southerly and northerly [latitudes] do not affect lunar eclipses. Sometimes seven months may elapse between two solar eclipses, such that at the first eclipse the Sun has not yet reached the tail and the Moon's latitude is northerly, and at the second eclipse [the Sun] has passed the head, so long as the [Moon's] latitude is still northerly. In lunar eclipses this is not possible, because in lunar eclipses the distance from the node must be less than it is in northerly solar eclipses.

[6] These rules apply to the northern habitations. In the southern ones, the opposite must be conceived. It is even possible for two solar eclipses to occur within the span of one month: one in northern habitations at a very northerly latitude, and the second in southern habitations at a very southerly latitude; but it is not possible in the same habitations. It often happens that solar and lunar eclipses occur within half a month [of each other]. This is all to be said in this chapter—God is all-knowing.

CHAPTER FOURTEEN
On Conjunctions and on Visibility and Invisibility of the Planets

[1] Since the position of a planet on the zodiacal orb is the endpoint of a line extending from the center of the World through the center of the body of the planet to the surface of the zodiacal orb; and since their true positions on the [zodiacal] equator in terms of degrees are where the latitude circle at the endpoint of the [above-mentioned] line intersects the zodiacal equator (if the planet has latitude, otherwise, at the endpoint of the line on the zodiacal equator), therefore, any two plan-

ets that are on one latitude circle, or one latitude circle passes through the two endpoints of their lines, will come together at the same degree on the [zodiacal] equator. This situation is called conjunction. The most complete conjunction occurs between two planets that are on two [different] orbs but coincide in latitude on the same side [of the zodiacal equator], such that one line extending from the center of the World passes through the center[s] of both planets. Such a situation is called latitudinal conjunction. Due to the parallax, for planets below the orb of Mars, it sometimes happens that a latitudinal conjunction occurs but, as observed, one does not occult the other, or one occults the other and yet there is no latitudinal conjunction.

[2] Since the rays of the Sun obscure the planets, their conjunctions with the Sun are not perceptible, except [in the case of] the solar eclipse, which is due to its conjunction with the Moon. What Abū ʿAlī Sīnā [Avicenna] said, that "I saw Venus like a black spot on the [Sun's] face," is possible, because when Venus is at epicyclic perigee, its diameter is about 5′ or more, and the Sun's diameter is 32′; thus, a sixth of the Sun's diameter is obscured by the body of Venus. There is no doubt that the rays of Venus and any other planet cannot vie in strength and dominance with the rays of the Sun, since, even if the whole body of the Sun were eclipsed and nothing more remained of the amount of the planet's body, the World would still be much brighter than the night with several thousand stars. Therefore, that amount of the Sun['s body] that is covered by Venus is not comparable with the rest of the body with respect to rays and light. For this reason, it appears like a black spot on its face. The purpose of this explanation is that there are many people who, out of ignorance, deny and object when they hear this.

[3] As for visibility and invisibility, they vary among the [wandering and fixed] stars: first, due to the smallness and largeness of the star's body; second, due to its ascendancy or deficiency of light; third, due

to different periods of rising and setting; fourth, due to the latitude [and] the declination [of the celestial body] in a direction opposite the locality (*ufuq*); and fifth, due to fast and slow speeds.[1] This latter [sic] is the main reason, to some extent, that the star Canopus is invisible for nearly half the year, while the star Vega, for example, is never invisible.

[4] Of the wandering planets, none appears earlier than Venus while retrograding and both its latitude and declination are northerly, whereby it does not remain invisible for more than two days in this situation. In some localities, Venus can be seen both morning and evening on a day of combust; hence, it will not be invisible [on that day]. During retrogradation, Venus has larger size, more light, and greater latitude, which do not occur for other planets. Furthermore, the planet Venus during its direct motion and the planet Mars have the longest periods of invisibility among the planets, since their speed is close to that of the Sun, and it takes longer for them to be elongated from it; their bodies in that state appear extremely small because they are at apex. As for the invisibility of the Moon, it is due to the vanishing of its light, as we have said. Its period of concealment is not less than two days or more than three. In these climes, it is usually the case that [the Moon] becomes visible when its altitude at sunset is [at least] 8 degrees, or when [the time] between its setting and sunset is four-fifths of an hour.[2] This is all that is to be said in this chapter, and with this we conclude Book II—God is the best granter of success and provider of succor.

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1. A, F, G, N: fourth, due to fast and slow speeds; and fifth, due to the latitude and the declination in a direction opposite the locality.

2. Here we follow Giahi Yazdi, who provides an explanation for how Ṭūsī arrived at these parameters for lunar visibility (Hamid-Reza Giahi Yazdi, "Naṣīr al-Dīn al-Ṭūsī on Lunar Crescent Visibility and an Analysis with Modern Altitude-Azimuth Criteria," *Suhayl* 3 (2002-3): 231-43, on 240-41).

BOOK III

On the Configuration of the Earth, and the Variability of the Circumstances of Locations on It with Respect to the Changing Positions of the Upper [Bodies]

CHAPTER ONE

On the Configuration of the Earth, and a Brief Exposition of Its Circumstances

[1] We have said previously that the Earth is spherical in shape, and is situated in the middle. Thus, its center is the center of the World and the center of heavy bodies. Circular motions are around it, and rectilinear motions are toward it or away from it.

[2] If it were not for heavenly causes necessitated by Divine providence—grand be His praise—water would surround the [Earth] completely, because earth is heavier than water and no part of the Earth would be uncovered. However, due to certain causes ordained by the Creator—grand be His name—it was necessitated that some of its surface be uncovered and adjacent to the air so that animals dwell on it. Water and earth are mingled, and altogether have become analogous to one sphere.

[3] Some scholars have said that the reason part of the Earth's surface is uncovered is that the Sun is closer to the Earth in the south and farther away in the north, since the Sun's apogee is in the north and its perigee in the south. When it is closer, its body appears larger and therefore its rays are stronger and its heat, necessitated by its rays, is more. [Now], it is the characteristic of heat to draw wetness to itself, as can be observed in a lamp when it draws oil to itself. Therefore, the excess of heat in the south draws water in that direction so that the northern side is left uncovered. On this basis, as the apogee of the Sun is displaced from one direction to another, the settled world will move too. Although this argument is satisfactory for explaining the reason why the northern hemisphere is characterized by settlement, it is not satisfactory for [explaining] why one of the two northern quarters is characterized by [settlement] rather than the other—and knowledge is with God.

[4] In general, the part of the Earth that is uncovered is considered to be about one quarter and is called the populated quarter. This quarter is not completely settled, but rather within this quarter there are seas in its localities and around them, as well as deserts and mountains, and some locations that are too hot or too cold for the human race to live there.

[5] As the equinoctial circle divides the visible surface of the Earth into two halves, a circle will occur on its visible [surface] that is like an equator for the terrestrial sphere. That circle is called the equator and divides the Earth into two hemispheres, one northern and the other southern. If another circle is imagined that is at right angles to the first circle and passes through the two poles of the first circle, each half is then divided into two halves. Each of these four divisions is then a quarter of the surface of the Earth. The populated quarter is one of the two northern quarters.

[6] The length of each quarter is half a great circle, and its width is one quarter. Now if two persons were to stand upright at the two extremities of the populated quarter, they would be at either end of a diameter of the Earth, and their feet would be aligned with each other; both would have one horizon circle. However, the visible half of the [celestial] orb would be the invisible half for one of them, and the visible half for the other. Then, if one divides each of these [two] circles in accordance with the celestial degrees, the length of the populated quarter will be 180 degrees and its width 90 degrees. Of this 90 degrees, the amount up to the complement of obliquity, which is 66 degrees and a fraction, has the possibility of habitation. Beyond that up to 90 degrees cannot be inhabited because of the extreme cold due to the distance of the Sun from the zenith.

[7] Therefore, the length of the settled world is 180 degrees, and its width is 66 degrees and a fraction. The sea encompasses most of this expanse, and that sea is called the encompassing sea. Even in the midst of this settled world there are many seas, some of which are connected to the encompassing sea, some not connected.

[8] Among those [seas] that are connected, the largest is the Sea of Oman (*ʿUmān*), which is also called the Sea of Persia (*Fārs*) and also the Sea of India (*Hind*). It has opened [a passage] through the land from the east to near the western boundary. The connection of this sea to the encompassing [sea] is in the east. The length of this sea from the east to where it reaches the western boundary is 2,660 parasangs, and its width is 900 parasangs, 330 parasangs of which is north of the equator, and the rest is to the south. The equator passes through most of this sea.

[9] Four inlets of this sea extend into the middle of the [settled] world:

[10] The first, which is in the west, is called the Gulf of Barbary (*Barbar*), which is at the border of the Barbary. The length of this inlet in

the north is 160 parasangs, and its width is 35 parasangs.

[11] The second inlet is called the Red Gulf. Its length in the north is 460 parasangs and its width 200 parasangs. Where it narrows, its width becomes 60 parasangs, and at that point it is called the Sea of Clysma (*Qulzum*). Clysma is a town on the shore. It is also called the Lingua Maris (*Lisān al-baḥr*).

[12] The third inlet is called the Persian Gulf, and Basra (*Baṣra*) is on its shore. Fārs and Kirmān are contiguous to it. The length of this gulf is 460 parasangs, and its width is 180 parasangs. Between this gulf and the Red Gulf is [a distance of] 500 parasangs, all of which is Arab provinces. The Tigris (*Dijla*) and Euphrates (*Furāt*), which come from the mountains of Anatolia (*Rūm*) and Syria (*Shām*), spill into this sea. The borderline of this gulf is the land of Sind, and there many rivers discharge into it. There are many islands in this gulf.

[13] The fourth inlet, called the Green Gulf, is in the land of India. It is about 500 parasangs long, and there are great islands in it.

[14] There is another sea, also connected to the encompassing [sea], called the Mediterranean Sea (*daryā-yi Rūm*). Its length, from Andalusia (*Andalus*) toward the east, is 1,600 parasangs. Between the edge of this sea and the Sea of Clysma is [a distance of] three way-stations. Where this sea joins the encompassing [sea], it is no more than three parasangs wide, [but] as it gets farther from the encompassing [sea], it becomes 200 parasangs [wide]. When it reaches the border of Syria, it is 260 parasangs [wide]. Many rivers from Byzantium (*Rūm*) discharge into this sea. Two inlets extend [into the land] from this sea: one is called the Gulf of Constantinople (*Qusṭanṭaniyya*), which is 160 parasangs long; and a second inlet [toward] the west, which is 70 parasangs long. The islands of the Greeks are in this sea. The Nile (*Nīl*) of Egypt, which comes from the south, from the Lands of the Moon, discharges into this sea.

[15] There is another sea, also connected to the encompassing [sea] toward the north, called the Sea of the Varangians [i.e., the Baltic Sea], which is a large sea.

[16] As for the seas that are not connected to the encompassing [sea], the largest is the Khazar Sea, also called the Caspian (*Ābiskūn*) Sea. Its length, from east to west, is 260 parasangs and its width is 200 parasangs. Several large rivers discharge into it, such as the Arax (*Aras*) River, which flows from Armenia; the Kura (*Kur*) River; the Volga (*Itil*) River, which comes from the region of the Bulgarians (*Bulghār*) and is larger than the Oxus (*Jayḥūn*); and the river called Sipīd-rūd.

[17] The other [unconnected to the encompassing sea] is the Aral Sea (lit., Khʷārazm Lake), into which discharges the Oxus of Khʷārazm, flowing from east of Balkh and [formed] from the conjunction of five large rivers, and the Jaxartes (*Sayḥūn*), which comes from Turkistān. The perimeter of this sea is 100 parasangs. Between it and the Caspian Sea is [a distance of] twenty way stations.

[18] In Syria there is a small sea called the Sea of Tiberias (*Ṭabariyya*), the size of the Aral Sea or smaller. In Armenia there is a lake in the area of Manzikert (*Malāzjird*). In sum, the details of the seas are numerous, and some of them are described in the books of roads and kingdoms (*masālik wa-mamālik*). This has been a general account of the populated quarter.

[19] Of the deserts, the Arabian Desert, the Maᶜbad Desert and the Khʷārazm Desert are quite well known. There are also other deserts, for on the periphery there is less settlement and more deserts.

[20] In his book *Geographia*, Ptolemy determined the width of the inhabited world to be $79+\frac{1}{4}+\frac{1}{6}$ degrees and said that of this total $16+\frac{1}{4}+\frac{1}{6}$ degrees is south of the equator, and 63 degrees is north [of it]. He determined the length of the settled world to be $177+\frac{1}{4}$ degrees. He said that there is more mention of northern habitations because this

is most of the settlement.

[21] The rationale for taking the starting point of the settled world with regard to longitude from the west is because the zodiacal sequence is this way. Ptolemy considered the beginning of the settled world at the Eternal Isles, which are islands in the Western Sea that were formerly settled. Some consider the beginning of the inhabited world at the shore of the Western Sea. There is a 10-degree difference between the two.

[22] The beginning of the settled world with regard to latitude is considered to be the equator. What lies to the south is said to be southern in latitude, and what is to the north is said to be northern in latitude. The positions of given localities are determined by longitude and latitude. Thus, the longitude of a given locality is an arc along the equinoctial between the meridian circle of either the Eternal Isles or the shore of the Western Sea and the meridian of the given locality. Hence, it is obvious that the farther one goes toward the north, the less the length of longitudinal degrees becomes, until at one point that is aligned with the [celestial] pole they all come together. The greatest length [of longitudinal degrees] is at the equator, because the meridian circles coincide with the declination circles. The latitude of a locality is an arc along the meridian [circle] between the equinoctial and the zenith of that locality. The length of latitudinal degrees is the same everywhere. Every locality whose longitude is less than 90 [degrees] is western, every [locality] whose longitude is greater than 90 [degrees] is eastern. Every locality whose latitude is less than 33° 12′ is southern, and any [locality] whose latitude is greater than this amount is northern.

[23] The Earth is divided into seven climes in width, such that the length of each clime is from east to west. Its width is an amount [that results] in a difference of half an hour in duration of daylight, ex-

cept for the beginning of the first clime and the end of the seventh clime, which are more than this amount because there is less settlement there.

[24] The beginning of the first clime is the equator. The equator starts from the south of western Sudan and passes north of the mountains known as the Mountains of the Moon, and north of part of the Zanj regions and the south of the harbor of Aden, and so on and so forth until [it reaches] Sarbuza Island, which is in the Green Sea, and the islands of Sarandīb to Dizkank, which is reckoned as [part of] China. The limit of the equator is at an island the Indians call Jamkūt. The midpoint of the equator is called the Cupola of the Earth. The midpoint of the first clime is where its latitude is 16° 27′ and its daylight is 13;0 [hours]. Among the localities of the first clime are western Sudan, part of the Barbary lands, Yemen, the lands of Abyssinia and the Zanj, and the Indian Islands to the border of China.

[25] The beginning of the second clime is where its latitude is 20°14′ and its day is 13 [hours] and 15 [minutes]. Its midpoint is where its latitude is 23°51′ and its day 13 [hours] and 30 [minutes]. Among the localities of this clime are parts of Egypt, Barbary and the Maghrib, most of the lands of the Arabs, Ḥijāz and Yemen, a side of Mukrān and Sind, and most localities of India.

[26] The beginning of the third clime is where its latitude is 27°12′ and its day is 13 [hours] and 45 [minutes]. Its midpoint is where its latitude is 30°22′ and its day is 14 [hours] and 0 [minutes]. Among the localities of this clime are parts of the lands of the Maghrib, Barbary and Ifrīqiyya, Alexandria, part of the lands of Syria, the Jazīra, Kūfa, Baṣra, Baghdād, most of Iraq of the Arabs, Ahwāz, Fārs, Kirmān, Sijistān, Zābul, Kābul, part of Indian lands, and a side of the Turkic [lands] and China.

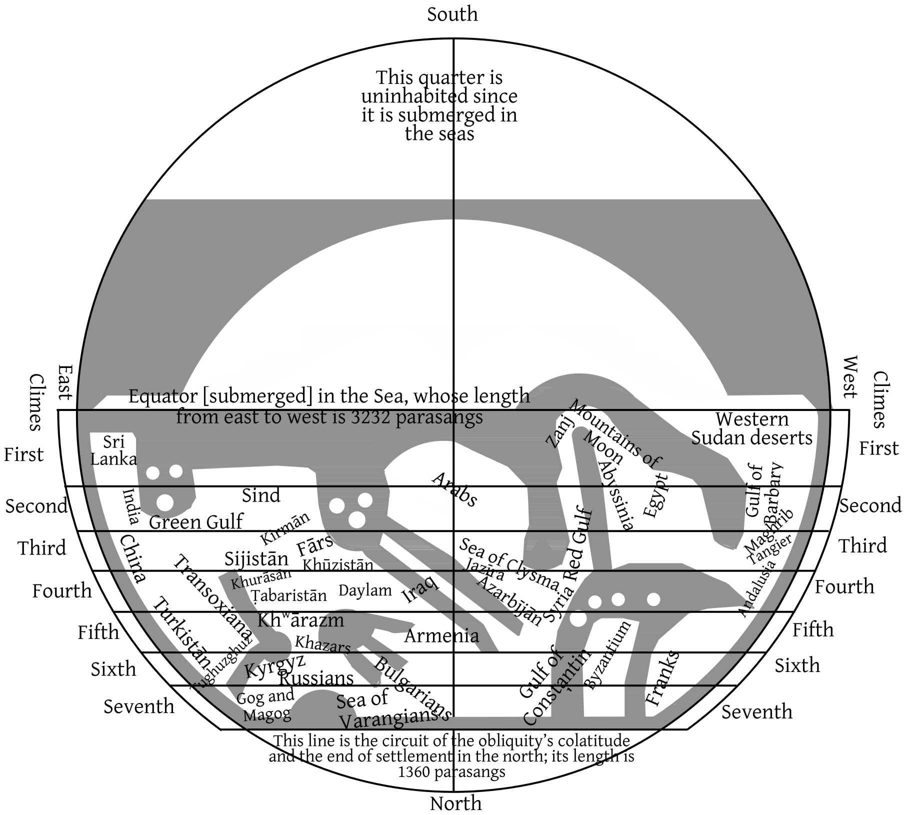
[27] The beginning of the fourth clime is where its latitude is 33°18′, and its day is 14 [hours] and 15 [minutes]. Its midpoint is where its latitude is 36°0′, and its day is 14 [hours] and 30 [minutes]. Among the localities of this clime are the lands of Andalusia, parts of the Maghrib, Anatolia, and Syria, most of Azerbaijan, the lands of Jazīra, Mosul, the towns of Iraq of the Persians, Qūmis, Daylam, Ṭabaristān, Gurgān, Khurāsān, Gīlān, Tibet, and a side of the Turkic lands.

[28] The beginning of the fifth clime is where its latitude is 38°35′, and its day is 14 [hours] and 45 [minutes]. Its midpoint is where its latitude is 40°56′, and its day is 15 [hours] and 0 [minutes]. Localities in this clime are part of Byzantine lands, Armenia, Khazar, Khʷārazm, Transoxiana, Farghāna, and part of the lands of Turkistān.

[29] The beginning of the sixth clime is where its latitude is 43°51′, and its day is 15 [hours] and 15 [minutes]. Its midpoint is where its latitude is 45°0′, and its day is 15 [hours] and 30 [minutes]. Localities of this clime are most of Byzantium, Khazar, Turkistān, and the lands of various [Turkic] groups.

[30] The beginning of the seventh clime is where its latitude is 46°51′, and its day is 15 [hours] and 45 [minutes]. Its midpoint is where the latitude is 48°32′, and its day is 16 [hours] and 0 [minutes]. Its end is the end of settlement. Localities of this clime are the lands of Slavs, the boundaries of the Turks, and Gog and Magog. In some of these localities it is so cold that its people have to spend six months in bathhouses.

[31] Some people place the beginning of the first clime where its latitude is 12°30′, and its day is 12 [hours] and 45 [minutes], and the end of the seventh clime where its latitude is 50°0′, and its day is 16 [hours] and 15 [minutes]. What lies above or below these places is not reckoned in the climes. This is the illustration of the Earth:



[Figure 1]

CHAPTER TWO

On the Characteristics of Localities That Are on the Equator

[1] In localities that lie on the equator, the equinoctial circle passes through the zenith; the intersection of the equinoctial and the horizon is at right angles; the equinoctial and the circle of the initial azimuth [prime vertical] are the same circle; and two poles of the equinoctial are on the horizon. The horizon circle bisects the day-circuits into two halves, one visible half and one invisible half. No part of the [celestial] orb is permanently visible or permanently invisible, but [the period of] invisibility of each part is equal to [the period of] its visibility. The ro-

tation of the orb [there] is wheel-like. The Sun in a year will pass twice over the zenith, once at the beginning of Aries and another [time] at the beginning of Libra. For one half of the year, shadows of objects fall toward the north and for one half toward the south; and shadows [cast at] the beginning of Cancer and beginning of Capricorn are equal. At the beginning of Aries and beginning of Libra, no shadow is cast [at all] at noon.

[2] There are eight seasons during the year. When the Sun reaches Aries and passes over the zenith, there is extreme heat and thus it is the season of summer. At the middle of Taurus, when [the Sun] is away from the zenith, autumn begins. At the beginning of Cancer, when [the Sun] is at the greatest distance from the zenith, it is winter. At the middle of Leo, when [the Sun] is headed back toward the zenith, it is [the beginning of] spring. Then once again summer begins at the beginning of Libra as the Sun reaches the zenith, and so on up to the full revolution. There are therefore eight seasons in a year, unlike other localities.

[3] One of the poles of the zodiacal orb is always above [the Earth], and one is [always] below the Earth. The periods of visibility and invisibility of the two [poles] are equal, except when the two equinox points pass through mid-heaven, at which time the poles of the zodiacal orb along with the equinoctial poles are on the horizon, and the solstitial colure coincides with the horizon circle.

[4] The Grand Master Abū ʿAlī Sīnā said that the [equator] is the most temperate of all localities, because the heat of day and the cold of night are balanced due to [their] equality, and because the Sun does not linger long over the zenith, because at the equinox point the Sun comes from one direction, headed in the other direction, and passes swiftly. [He also said] that the hottest localities are those that are parallel to the circuits of the head of Cancer or the head of Capricorn,

where daytime is very long, and the Sun lingers while being directly overhead. The most learned of the moderns, Fakhr al-Dīn Rāzī, raised objections to him and said that despite the fact that the Sun does not linger long over the zenith of those who dwell at the equator, nonetheless it never gets farther from their zenith more than 23 degrees and a fraction, and it always remains closer [than this amount to their zenith]. We see how hot the summer is in localities where the maximum altitude of the Sun is near this amount, which is its minimum altitude at the equator. An example [of a hot summer] is Khʷārazm, where the altitude at the beginning of Cancer is 71 degrees, with a 5-degree difference compared to [places with] the minimum altitude at the equator. Therefore, in places where the Sun's altitude is always more than this amount, the heat of winter is greater than the heat of summer in a place like Khʷārazm, because in Khʷārazm, the Sun's altitude is always less than its altitude at the beginning of Cancer, while on the equator it is more [than that]. What we see in the appearance and color of people of Zanj, whose habitations are near the equator, is a confirmation of this statement. Therefore, the equator is the hottest locality.

[5] When one considers these two statements [of Ibn Sīnā and Rāzī], it is obvious that "temperateness," in the sense of uniformity of conditions, is more at the equator than at any other locality. Due to this uniformity, [the difference in] quality of hotness [of weather] may not be so perceptible, because any perceptible thing is not felt so strongly when it is continuous, while any [perceptible] that comes suddenly after its opposite is greatly felt. Nonetheless, at the same time the extreme hot weather condition is also more there [i.e., at the equator], and therefore temperateness in the sense of balance between heat and cold is disproved for that locality. Therefore, based on the first interpretation, Master Abū ʿAlī's statement is correct, while based on the second interpretation, the statement of the latter scholar is also cor-

rect.

[6] The midpoint of the equator is called the Cupola of the Earth, where the longitude is 90 degrees. This is because west of that location is the western part of the settled world, and east of it is the eastern part of the settled world—God is all-knowing of the Truth.

CHAPTER THREE
On the Characteristics of Localities Whose Latitudes Are Less Than or Equal to the Obliquity

[1] Every locality that has latitude is reckoned among the "oblique horizons," because the turning of the equinoctial there is slanted. The right sphere does not occur anywhere other than the equator. When a latitude is assumed for a horizon, one pole of the equinoctial that is in the direction of the latitude stays above the horizon by the amount of the latitude, and the other pole is depressed [below the horizon by the same amount]. The day-circles whose distance from the equinoctial pole is equal to or less than the local latitude neither rise nor set, but rather those that are about the visible pole are permanently visible, and those that are about the invisible pole are permanently invisible.

[2] The year has four seasons as usual, except in locations whose latitude is less than the obliquity, where the Sun passes over the zenith twice, and thus the heat will be intense at these two times. Between these two times, when the Sun is closer to the visible pole in the direction of the solstice, there is a lessening in the heat of the air. During this time, the shadows of objects at noon fall toward the invisible pole, [whereas] during the rest of the year the [noon] shadow falls toward the visible pole. On those two days when the Sun passes over the zenith, there will be no shadow [at noon].

[3] The horizon circle divides the equinoctial into two halves, and other day-circles into two unequal parts. Of those [day-circles] that are in the direction of the visible pole, the visible part is larger, while of those [day-circles] that are in the direction of the invisible pole, the visible part is smaller. For any two day-circles equidistant from the equinoctial on its two [opposite] sides, the visible part of one is equal to the invisible part of the other. Therefore, when the Sun is at the beginning of Aries or Libra, daylight and nighttime are equal; when it is in the direction of the visible pole, the daylight of any part [on the zodiacal equator] is equal to the nighttime of the part diametrically opposite. For any two parts that are equidistant from the equinoctial on the same side, such as the beginning of Taurus and the beginning of Virgo, their daylights and their nighttimes are equal. Summer is longer in these localities because the Sun reaches the zenith twice; and the greater the local latitude, the closer the two points on the zodiacal orb that pass over the zenith, and the smaller the arc between them.

[4] The two poles of the zodiacal orb rise and set. The period of visibility of the pole that is not in the direction of the local latitude is as much [as the period] that the arc between the aforementioned two points that pass over the zenith is on the meridian. Its period of invisibility, and [the period of] visibility of the [other] pole that is in the direction of the local latitude, is the rest of the revolution. During the two times when those two points are at the zenith, the two poles of the zodiacal orb will be at two points [on the horizon], and the zodiacal orb's intersection with the horizon will be at right angles, i.e., the zodiacal orb is one of the altitude circles.

[5] As for those places whose latitude is equal to the obliquity, which are the hottest of all localities according to Master Abū ʿAlī, one pole of the zodiacal orb is permanently visible and one pole is permanently invisible. The Sun always passes in the direction of the invisible pole except for one day, when it reaches the beginning of the solstice in

the direction of visible [pole]. On that day, the [Sun] will be at the zenith. and no shadow falls. In all other times that a shadow falls, it is in the direction of visible pole. When that solstice is at mid-heaven and at the zenith, the zodiacal circle coincides with the circle of the initial azimuth. Therefore, in these localities, every point whose day-circle is between the two poles of the equinoctial and the zodiacal orb is permanently visible or permanently invisible, while all other day-circles have both [periods of] visibility and invisibility.

[6] These characteristics, and other ones that we are going to mention, are common for the northern habitations and southern habitations. The winter and summer of these two sides alternate, i.e., when it is summer in the north, it is winter in the south and vice versa. The other two seasons are like this too. Places in the south whose latitude is equal to the obliquity are hotter than the ones whose latitude is equal to the obliquity in the north, because of the [place of] apogee and perigee, as we have mentioned. Some of the practitioners of this science call such localities the "combust way," because there is no place hotter than those on the surface of the Earth. These are the characteristics of these localities—and God is all-knowing.

CHAPTER FOUR

On the Characteristics of Localities Whose Latitude Is Greater than the Obliquity, up to Where It Is Equal to the Complement of the Obliquity

[1] In these localities, all the day-circles of the zodiacal orb pass on one side of the zenith, and no part of the zodiacal orb reaches the zenith. The two poles of the zodiacal orb have two day-circles, one visible and one invisible. The [visible] pole in its day-circle has two extreme altitudes: one the highest altitude and the other the lowest altitude, at both of which times it will be on the meridian circle; the

invisible pole is the same. The Sun's greatest altitude will be at [one] solstice point, and its lowest [altitude] at the other solstice. All other conditions pertaining to day and night, the[ir] length and shortness, the circumstances of shadows, and the revolution of the equinoctial are as were stated in the previous chapter. The intersection of the zodiacal orb and the horizon is never at right angles.

[2] The greater a town's latitude, the higher the visible pole of the equinoctial and the larger the permanently visible and permanently invisible day-circles, up to where the town's latitude is equal to the complement of the obliquity. Thus, the permanently visible day-circle will be tangent to one solstice, and the permanently invisible day-circle will be tangent to the other solstice. The pole of the zodiacal orb reaches the zenith once every revolution, and that is when the solstice is tangent to the horizon. Thus, one solstice is permanently visible, and one solstice is permanently invisible. [Then] daylight increases until it is the entire day, and for one nychthemeron the Sun will not set, but will become tangent to the horizon at the point of intersection between the meridian and the horizon, and [then] it will rise again. Likewise, nighttime increases until [the day] becomes entirely night, and for one nychthemeron the Sun will not rise, but will become tangent to the horizon, and [then] it will go back [down]. Once every nychthemeron the horizon circle and the circle of the zodiacal orb coincide with one another, and in one stroke half of the zodiacal orb [i.e., its equator] rises, and the other half sets. Therefore, one half of the zodiacal [equator] rises in a nychthemeron, and one half in one stroke. If the visible pole is northerly, that half between Capricorn and Cancer will rise in one stroke; and if the visible pole is southerly, [then] the other half [will rise in one stroke]. The [celestial] parts that rise and set are those parts whose distance from the equinoctial is less than the obliquity. Other celestial parts neither rise nor set—God is all-knowing.

CHAPTER FIVE

On the Characteristics of Localities Whose Latitude Is Greater than the Complement of the Obliquity, up to Where the Latitude Reaches [Its] Maximum

[1] In these localities, the solstice point that is in the direction of the visible pole, along with two equal arcs on its two sides, are permanently visible. The two endpoints of those two arcs are the two points whose declinations are equal to the local colatitude. Those two points become tangent to the horizon along their day-circles and do not set. The other solstice point, along with two arcs that correspond in opposition to these two [above-mentioned] arcs, are permanently invisible. The two points that are those arcs' endpoints become tangent to the horizon and do not rise. Their point of tangency is the north or south point. [All] other parts of the zodiacal orb rise and set: one half in regular order, and one half in reverse order. That which rises in regular order, sets in reverse order, and that which rises in reverse order, sets in regular order. In northern localities the Arian arc rises in reverse order, and in southern localities the Libran arc.

[2] The altitude of the visible solstice has two extremes: one beyond which it does not go higher, and one beyond which it does not go lower. Describing a single cycle, this can easily be imagined. Whenever the visible solstice point is on the meridian circle, it is at the highest altitude, the invisible solstice point is below the Earth in the other direction, the two equinox points are on the eastern and western [horizon], and the visible pole of the zodiacal orb is at its lowest altitude on the meridian in the opposite direction of the visible solstice. Afterwards, when the visible solstice point heads west, the pole of the zodiacal orb on the other side [of the meridian] begins to increase in altitude; one equinox point sets, the [other] one rises, and the arc connecting them begins to rise and set sequentially. The rising place of each [zodiacal]

degree and the setting place of the point directly opposite it get farther away from the rising and setting places of the equinox.

[3] Whereupon the shift comes to the two opposite points [on the zodiacal equator], one of which comes to be tangent to the horizon but does not set, and the other comes to be tangent to the horizon but does not rise, these two points become tangent to the horizon at the two poles of the circle of the initial azimuth, the permanently visible point in the direction of the visible pole and the permanently invisible point in the opposite direction. The visible half of the zodiacal orb is toward the west, from north to south, and the other half, which is invisible, is directly opposite it. The places of intersection between the horizon and the zodiacal orb are the two poles of [the circle of] the initial azimuth. The visible pole of the zodiacal orb is toward the east, between the lowest and highest altitudes on the circle of the initial azimuth, the other [pole] being directly opposite to it.

[4] Then the endpoint of the arc that is tangent to the horizon on the meridian circle and is permanently visible rises from the horizon and begins to gain altitude obliquely in the eastern region. The arc that is connected to the [aforementioned endpoint] beneath the Earth begins to rise in reverse order such that every part that rises gets closer than the preceding part to the rising place of the equinox. The diametrically opposite point drops below the horizon, and the arc that is connected to it begins to set in reverse order, every part opposite another, whereupon the shift comes to the two equinox points to rise and set in reverse order, and the pole of the zodiacal orb will have reached the meridian circle at [its] highest altitude. The visible half of the zodiacal orb will be toward the north; the intersection of the zodiacal orb and the horizon will be at the two points of rising and setting of the equinox; the invisible half of the [zodiacal orb] will be facing [in the opposite direction] this half [i.e., the visible half]; the visible solstice point will be at its lowest altitude on the meridian; and the invisi-

ble solstice will be directly opposite it, at the closest possible situation with respect to the horizon.

[5] Afterwards, the arcs that are connected to the two equinoxes will be rising and setting in reverse order; the visible pole of the zodiacal orb begins to descend; and the solstice point begins to rise, until on the other side of the equinox point, when it will shift at the second point of the permanently invisible points. This point will be tangent to the horizon on the meridian, and the point directly opposite it, which is permanently visible, will be tangent to the horizon on the meridian in the other side. The visible half of the zodiacal orb will be on the eastern side, from north to south, and the invisible half in the opposite direction. The pole of the zodiacal orb will have reached the circle of the [initial] azimuth. Then the permanently visible point will rise above the Earth in the eastern direction, and the other point will drop [below the Earth]. The arcs that are connected to it will begin to rise and set in regular order. The rising and setting places of any part that rises and sets get closer to the east and west of the equinoctial, until when the shift is attained at the equinox point; then a complete revolution will have been made, and the first situation will have been returned to exactly.

[6] At such localities, daylight and nighttime are equal at the beginning of Aries and Libra. When the beginning of Aries has passed in northern habitations, and the beginning of Libra in southern habitations, daylight increases and nighttime decreases until it becomes all daylight. As long as the Sun traverses the permanently visible arc, daylight remains without night; night then reappears and increases until the [Sun] reaches the other equinox point, when daylight and nighttime become equal. Then nighttime increases and keeps increasing, until it is all nighttime. As long as the Sun traverses the permanently invisible arc, nighttime remains with no daylight; then once again daylight re-emerges and keeps increasing until it is equal to nighttime.

The shadow will be cast in all directions, but mostly in the direction of the invisible pole.

[7] When the local latitude reaches the limit, i.e., 90 degrees, the visible pole of the equinoctial will be aligned with the zenith, the other pole will be directly opposite to it, and the equinoctial circle will coincide with the horizon circle. The turning of the celestial sphere is in a spinning manner. None of the celestial parts rises or sets because of the rotation of the equinoctial; rather, half of the celestial sphere is permanently visible, and half is permanently invisible. The visibility and invisibility of the stars depends on the second motion: those whose latitude is less than the obliquity rise and set, while those whose latitude is more do not rise or set.

[8] For a period of six months the Sun is above and for a period of six months below the Earth. Therefore, every year will be a nychthemeron, with six months of daylight and six months of nighttime. Opposite that direction in which the perigee [of the Sun is found], i.e., in the northern direction inasmuch as the apogee is northerly, daytime is greater than nighttime; in the southern direction, nighttime is greater than daytime. Of the six months that are night, nearly seventy days is the period of dawn and nearly seventy days is the period of dusk, the darkness of nighttime being not more than forty days. The altitude of the Sun in such a locality can be no more than the obliquity. On the surface of the Earth there are no more than two spots with this characteristic, unlike the other characteristics whose type is specific to two circuits on the external surface [of the Earth]. In most of these localities the survival of animal species that are seen [elsewhere] is not possible.

[9] One may ask the practitioners of this discipline: [there are] three individuals in a location on the surface of the Earth, one stays and two travel. Of the travelers, one travels toward the east and returns to the

stationary individual from the west; the other travels toward the west and returns to the stationary individual from the east. All three have been counting the days. The easterner, for example, said: "Today is the hundredth day since we have left the stationary individual." The westerner said: "No, it has been a hundred and two days." The stationary individual said: "No, it has been a hundred and one days." Or the easterner said: "Today is Saturday;" the westerner said: "It is Monday;" the stationary individual said: "It is Sunday." None of them has made a mistake in counting. How can this be?

[10] Conceptually, this is [actually] true, since for the one who goes toward the east and returns from the west, one celestial revolution will be subtracted from his course because every day the Sun rises earlier for him, and so his nychthemerons get shorter than those of the stationary individual; and during a [complete] revolution those deficits add up to a day. The Sun rises later every day for the one who sets out toward the west and returns from the east, and his nychthemerons get longer than those of the stationary individual; those increments, distributed over the nychthemerons, add up to a revolution, which is one day. The stationary individual has his own standard [celestial] revolutions. These are the states of affairs of the locations of the Earth, which are dependent on the celestial conditions that have been mentioned—God the Almighty is all-knowing.

CHAPTER SIX

On the Co-ascension of the Zodiacal Signs, Which Is the Rising of the Parts of the Equinoctial with the Parts of the Zodiacal [Orb]

[1] For every location, the rising of the parts of the zodiacal [orb] with the parts of the equinoctial is different from any other location because of the declination, as has been mentioned previously. The

amount of the equinoctial that rises with a given amount of the zodiacal orb is the co-ascension of those parts. In localities that are on the equator and hence have no latitude, the horizon of that locality is one of the declination circles, since it passes through the two poles of the equinoctial. Those localities are called straight horizons, and the co-ascension of those localities is called the co-ascension of the right orb, the co-ascension of the erect sphere, and the co-ascension of the equator.

[2] When an equinox point is on the horizon, the other [equinox] point is on the horizon in the other direction, and the two solstices are on the meridian circle. Thereafter, when an arc of the zodiacal orb rises, the arc on the equinoctial that rises is less than that, because the equinoctial intersects with the horizon at right angles, and the zodiacal orb at acute and obtuse angles. This situation continues until the equinox point reaches the meridian and the solstice point is on the horizon, at which time a quarter of each of these two equators will have risen completely and both circles intersect with the horizon at right angles. Thereafter, equal arcs of the zodiacal orb rise with unequal arcs of the equinoctial, as in the first quarter but in reverse order. For example, parts of Cancer remain in the sequence [of the signs], while parts of Gemini are in counter-sequence. [This continues] until another quarter rises and the equinox point reaches the horizon. All co-ascensions are as has been described for these two quarters.

[3] The co-ascensions of any four arcs equidistant from the two equinox points, such as the first ten degrees of Aries and the first ten degrees of Libra and the last ten degrees of Pisces and the last ten degrees of Virgo, are equal, as the co-ascensions of any four arcs equidistant from the two solstice points are equal. The initial point of the co-ascensions is taken to be the vernal equinox, [although] some [place it] at the beginning of Capricorn for another purpose, which shows up in practical [applications]. Since the passage of the parts of the zodiacal

[orb] across the meridian circle is similar inasmuch as the meridian circle is one of the declination circles, indeed being one of the horizons at the equator, in all localities the co-ascensions for the equator are used.

[4] As for the co-ascensions of the oblique horizons, which are those that have latitude and in which the turning [of the orb] is slanted, when the point of intersection is on the horizon, the solstice point is on the meridian. Then the point of intersection rises. If the arc of the zodiacal orb that rises is northerly, in northern habitations what rises from the equinoctial is less than that [arc], because the angles between the zodiacal orb and the horizon are more acute than angles between the equinoctial and the horizon; in southern habitations, it is the opposite. If that arc is southerly, in northern habitations what rises from the equinoctial with that [arc] is more than it; in southern habitations, it is the opposite.

[5] There are different rules for the quarters in these regions, because at the time a quarter of the zodiacal orb rises, not a full quarter of the equinoctial will have risen, if the arc of the zodiacal orb is in the direction of the local latitude; or, if it is in the opposite direction, more than a quarter will have risen. Nonetheless, the rules for the two halves of the zodiacal orb are the same, except that in one half it is in the sequence [of the zodiacal signs] and in the other it is in countersequence. Therefore, any two arcs equidistant from the equinox point have the same co-ascensions, and the co-descension of any sign is equal to the co-ascension of that sign's facing counterpart, because with [the rise of] any [zodiacal] sign, one will set. Therefore, the co-ascensions of the [zodiacal] signs in the north are exactly the same as the co-descensions of the [zodiacal] signs in the south, and the co-descensions of the [zodiacal] signs in the north are the co-ascensions of the [zodiacal] signs in the south.

[6] When the local latitude is equal to the complement of the obliquity, one half of the orb whose midpoint is the equinox point will rise in one stroke and there will be no co-ascension for it; in the other half, the entire equinoctial will rise. In localities whose latitude is greater than the complement of the obliquity, any two arcs that are permanently visible or permanently invisible will have no co-ascensions, but otherwise any two arcs will have co-ascensions, one in reverse order and the other in regular order. When the local latitude reaches the limit, the rotation of the orb being in a spinning manner, the co-ascensions will become completely baseless, since there will be no rising nor setting, and the equinoctial and the horizon will coincide.

[7] Some call the parts of the equinoctial "degrees," and some "units of time," because time is determined by the amount of its motion. This has been a description of the co-ascensions of the [zodiacal] signs—God is all-knowing.

CHAPTER SEVEN

On Determining the Equation of Daylight and the Ortive Amplitude in the Localities

[1] The arc on the horizon between the ascension of any part [of the zodiacal orb] and the ascension of the equinoctial is called the ortive amplitude of that part. It is obvious that the maximum ortive amplitude at the equator is equal to the obliquity. In other localities, the ortive amplitude increases with the local latitude, until the local latitude reaches the complement of the obliquity, and the ortive amplitude of a quarter of the orb is attained. The ortive amplitude of every quarter of the orb is equal to that of another quarter, one in the sequence [of the zodiacal signs] and the other in counter-sequence. The ortive amplitude of the two northern quarters is like that of the two southern quarters. [Also,] the ortive amplitude of any part is like the

occasive amplitude of its facing counterpart.

[2] The equation of daylight, which is the difference between the meridian of any part and the meridian of the equator, is an arc of the day-circle of that part, to which the equation of daylight is related, between the horizon circle and the declination circle that passes through the two poles of the meridian. Therefore, in the half that is in the direction of the visible pole, that arc will be above the Earth, and in the other half it will be below the Earth. That which is toward the west will be equal to what is toward the east, and that which is above the Earth will be equal to what is below the Earth, like equal declinations, i.e., parts whose distances from the two equinox points are alike in both directions.

[3] From the declination circle that passes through the two poles of the equinoctial, and from the horizon circle, and from the day-circle, a triangle will occur above the Earth or below the Earth: one side of that triangle will be the declination of that part to which the day-circle belongs; one side the ortive amplitude; and one side the equation of daylight. The arc along the equinoctial that rises with the aforementioned arc along the day-circle is also called the equation of daylight; and it is an arc along the equinoctial between two declination circles, one passing through the rising place of the equinoctial and the other passing through the rising place of that part. This arc will be below the Earth for parts whose declination is in the direction of the visible pole; and above the Earth for those whose declination is in the direction of the invisible pole. For the parts whose declination is in the direction of the visible pole, the equation of daylight is added to a quarter revolution, which then becomes half the arc of daylight; for those whose declination is in the other direction, it is subtracted from a quarter revolution, which then becomes half the arc of daylight. Half of the arc of daylight is an arc that rises for half the period of visibility of any part, being one half of the visible segment of any day-circle; the arc

of night is the complement of this arc up to half a revolution—God is all-knowing.

CHAPTER EIGHT
On Determining Degrees of Transit, Rising, and Setting

[1] The degree of transit of any [celestial] part is that degree of the zodiacal orb that transits the meridian along with that part. The degree of rising is that degree that rises with [that part], and the degree of setting is that degree that sets with it. When that part has no latitude, the degree of true position will be exactly the degree of transit, rising, and setting. However, if that part has latitude and is also on the solstitial colure, it transits the meridian circle at the degree of true position. If it is not on the solstitial colure, its degree is one of two kinds: it is either between the first of Capricorn and Cancer or between the first of Cancer and Capricorn.

[2] If [the part] is in the first half, at the time its degree [of transit] crosses [the meridian], the pole of the zodiacal orb is in the direction of the local latitude in the western half. Therefore, one half of the latitude circle that crosses through two parts of the zodiacal orb on the meridian is northwesterly, and one half is southeasterly. This being so, parts whose latitudes are northerly cross the meridian before the degree [of transit], and parts whose latitudes are southerly transit the meridian after the degree.

[3] In the second half [of the zodiacal orb], it will be the opposite: the pole of the zodiacal orb is on the eastern side, and [one] half of the latitude circles crossing parts on the zodiacal orb that are on the meridian is northeasterly and [the other] half is southwesterly. Thus, those whose latitudes are northerly cross after the degree, and those whose latitudes are southerly cross before the degree. It is likewise at the equator: of those between the first of Capricorn and Cancer, the

northerly cross before the degree and the southerly after the degree; for those between the first of Cancer and Capricorn, it is the opposite.

[4] As for degrees of rising and setting, wherever the pole of the zodiacal orb is on the horizon, at that time the degree of all that rise or set is its degree of rising or setting. When one pole of the zodiacal orb is above the Earth, the rising of every star in the direction of that pole is before the degree, and [its] setting is after the degree. This is because from that direction when a latitude circle is imagined passing through the two points of the ascendant and descendant, the half that is above the Earth will be in the direction of the visible pole; thus, every part in that direction either will have risen before the degree or will not yet have set. The half below the Earth will be in the direction of the invisible pole, and parts in that direction either will have set before the degree or will have not yet risen. The rising and setting of parts at the equator are like their transits of the meridian, because the horizon at the equator is one of the meridian circles—God is all-knowing.

CHAPTER NINE
On Determining Day and Night, Dawn and Dusk, Unequal and Equal Hours, etc.

[1] Since day and night arise from the motion of the equinoctial, and the Sun moves in the opposite direction, therefore the period of one nychthemeron, which is from the Sun reaching a given point until it reaches that point a second time by the diurnal motion, is one revolution of the equinoctial plus the Sun's motion. Since the Sun's motion is not uniform, and the rise of parts on the zodiacal orb with parts of the equinoctial are not in conformity, differences occur in the periods of nychthemerons from two aspects: one due to the difference in the diurnal courses of the Sun, and the other due to the difference between equal degrees and degrees of co-ascension. Therefore, a mean

day is the amount of a revolution of the equinoctial with the addition of the Sun's daily mean, while a true day is the amount of one revolution plus the Sun's [actual] course during that revolution in terms of the equinoctial co-ascension. The difference between the mean and the true, which is compounded of the two aforementioned differences, is called the equation of the nychthemeron. Although this difference is not perceptible over one day or two days, it can be perceived over many days.

[2] The maximum difference between the Sun's mean and its true position is in the amount of the equation. Since the equation is additive in one half and subtractive in the [other] half, the maximum difference between mean and true days is twice the equation. The maximum difference between equal degrees and the degrees of co-ascension is $2+\frac{1}{2}$ degrees; since this is sometimes additive and sometimes subtractive, the maximum difference between true and mean days is therefore 5 degrees. Only rarely, however, are these two differences completely compounded together, for when one reaches maximum the other is usually less than maximum.

[3] As for the difference that arises from the equation of the Sun, it is subtractive in one half of the orb where the apogee is at the midpoint of that half, and additive in the other half. The equal degrees are greater than the degrees of co-ascension in the two quarters whose midpoints are the vernal and autumnal equinoxes, and smaller in the other two quarters. Thus, since at this time the Sun's apogee is at the end of Gemini, both increases come together in the quarter whose midpoint is the winter solstice.

[4] Since one specific day must be assumed when the mean and true coincide, so that every difference can be determined with respect to that day, and [since] for any given part other than the two ends of this quarter the equation is sometimes additive and sometimes subtrac-

tive, the practitioners of the discipline of the stars have assumed that specific part to be in Aquarius, so that the equation of the nychthemeron is always subtractive from mean days and additive to true days. Had they assumed it to be a part in Scorpio, it would have been the other way around. When one revolution of the Sun is completed, mean and true days will be at their original state, and the difference will be eliminated. Such is the equation of the nychthemeron.

[5] The beginning of a nychthemeron would naturally be placed at the beginning of daytime, except if mathematicians take it to be from the beginning of daytime or the beginning of nighttime, then another difference would be added to the equation of the nychthemeron. That difference [comes from] co-ascensions, which differ by localities, and add to or subtract from the beginning of day and night the amount of half the increase or decrease in the length of the day, [which is] due to the difference in the divisions of the day-circles. On the other hand, since the beginning of a nychthemeron is made to be the passage of the Sun across a circle by which all day-circles are divided with an equal proportion, such as the meridian circle, this difference disappears. Thus, for this reason mathematicians take the beginning of a nychthemeron to be noon and calculate the true position according to that time. Those who are not occupied with such calculations make the beginning of a nychthemeron the beginning of daytime, as do the Persians. The Arabs, on the other hand, because the beginning of their months is according to crescent visibility, make the beginning of a nychthemeron to be the beginning of nighttime.

[6] The beginning of daytime is when the Sun reaches the horizon circle and not the rise of dawn. The beginning of nighttime is when the Sun reaches the horizon and not the setting of dusk. As for dawn, which is the light of the Sun when it approaches the horizon, it has different circumstances due to the shape of the Earth's shadow, as it is in the form of a circular cone, as has been explained. So, when the Sun

is near the nadir, the apex of the cone is near the zenith; thus because of the accretion of darkness, the light of the Sun, which is at the edges of the Earth and encompasses the shadow cone, is not perceptible. Afterwards, as the Sun approaches the horizon and the cone inclines toward the west, an elongated light appears from a narrow slit on the side toward the east. That light is above the horizon, because the lines extending from the observer's position, i.e., the surface of the Earth, to the horizon are longer than those extending to the surface of the cone from above the horizon, as has been established by geometrical proofs. Therefore, first dawn is elongated, and its base, which is connected to the horizon, is dark; hence it is called "false." Afterwards, as the cone inclines more, the horizon becomes illuminated and the light spreads, which is "true dawn." After that, the horizon turns red, on account of the intensity of light, until the Sun rises. The situation of dusk is similar, only in reverse: first redness, then extensive whiteness, then elongated whiteness.

[7] By testing and observation, it has been determined that the beginning of dawn and the end of dusk occur when the altitude of [the degree directly opposite] the Sun or the Sun's depression below the horizon reaches the amount 18 degrees. Therefore, in habitations whose colatitudes are less than the obliquity plus 18 degrees, their dawn will be continuous with their dusk, and dusk continuous with dawn, when the Sun reaches degrees where the sum of the declination and the local colatitude surpasses 72 degrees. Since the Sun's depression being 18 degrees below the Earth is like its altitude in its [directly opposite] degree above the Earth, then in the oblique horizons the period of dawn and dusk is greater in the one half of the zodiacal orb that is in the direction of the local latitude than is the period of dawn and dusk in the other half. For example, in the fourth clime the longest period of dawn, which amounts to two hours, is at the beginning of Cancer, and the shortest period, which is one hour plus a fraction, is at the begin-

ning of Capricorn.

[8] As for the hours of daytime and nighttime, they are of two sorts: one being equal, the other unequal. Seasonal [temporal] hours are also unequal. Equal hours are those that divide a nychthemeron into twenty-four equal divisions, each division being an hour. Therefore, when daytime of a [nychthemeron] is longer, the number of daytime hours increases; and when daytime is shorter, the number of hours decreases. The measure of hours is always equal, which is 15 degrees plus a bit of a revolution of the equinoctial.

[9] Seasonal [temporal] hours are those that divide the daytime, whether it be long or whether it be short, into twelve divisions, and so too the nighttime, each division being called an hour. Therefore, the measure of daytime hours differs from the measure of nighttime hours; the measure of one daytime hour along with the measure of one nighttime hour together equal the measure of two equal hours. In habitations at the equator there is no difference between equal and unequal hours—and God is all-knowing of the Truth.

CHAPTER TEN

On Determining the Year, Month, Calendar, Intercalation, and Their Like

[1] The basis of the month is from the visibility of the crescent to the full Moon until once again becoming imperceptible at the new Moon. Since this [cycle] is completed in approximately thirty days, and a year is completed in approximately twelve of these cycles, therefore the yearly cycle has been set to be twelve months and the monthly cycle to be thirty days. This situation also conforms to the twelve zodiacal signs and [their] having thirty degrees each. Since the best known of the planets and celestial bodies are the two luminaries, most nations

have considered the cycles of one of these two luminaries for setting up their months and years, with some having taken both into consideration. A year then is either solar or lunar, each of which may be either true or conventional.

[2] A solar year is [a period of time during] which the Sun leaves a [particular] point on the zodiacal orb and makes a full revolution, reaching that [same] point again. This occurs over a period of approximately 365+$\frac{1}{4}$ days.

[3] A true solar [year] is based on the [actual] revolutions of the Sun, not the number of days and months; one such is the Malikī year, wherein New Year's Day [Nawrūz] is set when the Sun reaches Aries. The months of this calendar are conventional, though if months were also based on the beginning of the zodiacal signs, the months would be true [solar months]. Since, however, the convention is to set months to be thirty days each, five days are left over, and they are called the stolen five [i.e., the epagomenal days]. Every few years, the fractions in excess [of 365] add up to a day, which is called the intercalary day.

[4] A conventional solar year is based on a number [of days] close to the true amount, like [that of] the Byzantines [Rūmiyān], who take the year to be exactly 365+$\frac{1}{4}$ days and thus every four years one intercalary day is added. They [i.e., the Byzantines] have distributed the stolen five days at the beginnings of the months, and therefore their year is never more than 366 days or less than 365 days. Some of their months have thirty [days] and some thirty-one. Since seven months have thirty-one [days], Shubāṭ has been set to be twenty-eight [days] or, during a leap year, twenty-nine. There is no rationale for such a set up. The Persians have set their year to be exactly 365 days, so that they do not have to take intercalation into consideration. [Their] months are each thirty days, with the stolen five [days added] at the end of the year. In olden times, they had an intercalary month every 120 years. Therefore, their

years, with that intercalation, became equal to the Byzantine years.

[5] A lunar [year] is [defined as] the Moon's reaching the Sun twelve times, which is achieved over approximately $354+\frac{1}{5}+\frac{1}{6}$ days. Each cycle of these twelve cycles is a month. This state of affairs will be a true [lunar month] if one of the Moon's positions vis-à-vis the Sun is made the starting point; when it reaches that position [again], it is counted as one month, as with the Arabs, who have established the visibility of the crescent as the beginning of the months. Their calendar is true lunar, with regard to both years and months.

[6] A conventional [lunar calendar] would take into consideration days and months, not consideration of the Moon's motion. This is the custom of those practitioners of arithmetic and astronomers, who take the year to be $354+\frac{1}{5}+\frac{1}{6}$ days; and from the beginning of Muḥarram, they take one month to be thirty days, and one month to be twenty-nine days until the end of the year. To account for the $\frac{1}{5}+\frac{1}{6}$, in every thirty years they intercalate eleven times and consider Dhū al-Ḥijja to be a full thirty days. This calendar is conventional with regard to both months and years. The years and months of the Jews are composed of lunar and solar. Their months are conventional, and every three years or two years an intercalary month is added so that the cycles of their years conform to the cycles of solar years. This convention is close to that of conventional solar years.

[7] It is for every nation to establish its own convention according to [its] judgment and want. Every nation sets the starting point of its calendars to be the beginning of a people or dynasty, or the occurrence of a great and famous event to which they relate their years and their months, such as the Arabs [who have chosen] the emigration of the Prophet—peace be upon him—the Byzantines the kingship of Alexander, son of Philip, and the Persians [the kingship of] Yazdgird, son of Shahryar. Precise knowledge of the principles of calendars and con-

verting them to one another belong to books of praxis. This much on the knowledge of the true sense of the years and months is sufficient.

CHAPTER ELEVEN
On Understanding Shadows and Their Circumstances with Respect to Altitudes

[1] From the preceding chapters it has been ascertained that the [solar] altitude at noon, which is the maximum altitude of the Sun, is the amount of the Sun's declination plus local colatitude if the Sun is in the direction of the visible pole of the equinoctial, or the amount of the excess of local colatitude over the declination if it is in the other direction. [It has also been ascertained that] there is a shadow corresponding to every [solar] altitude. Just as the upper limit of the [Sun's] altitude is 90 degrees, and its lower limit is when the luminary is on the horizon, likewise, the upper limit of a shadow is infinity, and its lower limit is when there is no shadow at all [i.e., at noon]. Other shadows [in between] are in proportion to the altitude [of the Sun].

[2] The shadow of a gnomon is a line drawn from its base to the end of a line from the body of the luminary through the gnomon's tip to the surface on which that gnomon is standing erect. The hypotenuse of the shadow is a line from the gnomon's tip to that surface, as a part of the aforementioned line. Therefore, the height of any gnomon, the shadow, and the hypotenuse of the shadow all three lines [together] form a right triangle, the subtense of the right angle being the hypotenuse of the shadow.

[3] Gnomons are either perpendicular to the plane of the horizon or [perpendicular] to a plane that is perpendicular to the horizon, i.e., they are parallel to the horizontal plane. Then, if the gnomons are parallel to the horizon, their shadows are called primary shadows; they

start at sunrise and reach their [upper] limit when the Sun is at the zenith. If the gnomons are perpendicular to the horizon, their shadows are called secondary shadows, which reach their [upper] limit at sunrise, and when the Sun is at the zenith it is diminished. Thus, the [lower] limit of one shadow is like the [upper] limit of the other shadow. For this reason, the primary shadow of any altitude is equal to the secondary shadow of the complement of that altitude. Because most shadows are secondary, and those are more visible, such shadows are called regular shadows, while the primary shadow is [called] a reversed shadow. The primary shadow is used in astronomical operations, and its measuring scale is taken to be 60 degrees, while some take it to be 1 degree. The secondary shadow is used in determining time, and its measuring scale is divided sometimes into seven divisions, or into six and a half divisions, that are called "feet"; sometimes into twelve divisions called "digits"; or sometimes into sixty [divisions] called "parts."

[4] In the fourth clime, the shortest noon shadow is the one cast at the beginning of Cancer, and the longest shadow is at the beginning of Capricorn. In other climes, it is according to altitudes, a bit of which has been explained in the preceding chapters—God is all-knowing.

CHAPTER TWELVE
On Determining the Meridian Line and Azimuth of Localities

[1] The meridian line is a line assumed on the surface of the Earth parallel to the [celestial] meridian circle. The line perpendicular to that line, which necessarily is parallel to the prime vertical [lit., circle of the initial azimuth], is called the east-west line. There are many ways to determine the meridian line, but the most famous of all is the Indian Circle. It is [constructed] such that on level land a pole is inserted.

Then its perpendicularity is checked by drawing a circle at the center of which the pole stands erect. Then the distances of three positions on the circumference of the circle from the vertex of the pole are measured. If they are equal, the pole is erect at right angles; otherwise it is inclining to one side. Then, at the beginning of the day, when the shadow has begun to decrease and intersects the circle, one watches to see at which point it enters the circle. At the end of the day, [one watches to see] at which point it exits. One draws a straight line between those two points, then extends a straight line from the center of the circle to the midpoint of that line. That line is the meridian [line], and the line perpendicular to it is the east-west line.

[2] If desired, one can mark the shadow of that gnomon at two times of equal altitudes on both sides of midday. From the base of the gnomon, one separates equal amounts from both shadows and draws a line connecting the two separation [points] so as to form an isosceles triangle from the two sides of the shadows and the produced line. Then from the midpoint of this line, a line is drawn to the base of the gnomon, which is the meridian line. Then that line, as we have said, corresponds to the meridian circle, and the east-west line corresponds to the prime vertical. When the point of intersection of these two lines is made a center [about which] a circle of any desired distance is drawn, that circle corresponds to the horizon circle.

[3] As for the azimuth of localities, it is an arc of the horizon circle between the north or south point and the intersection of the horizon circle with one circle of the altitude circles passing through the zenith point of another town. When two towns are longitudinally equal but latitudinally different, those two towns have no azimuth with respect to one another, the meridian line being their azimuth. This means, in the town whose latitude is less, one simply should face the north point [to face the place of greater latitude], while in the other town [of greater latitude one should face] the south point. As for two towns

whose latitude is the same but whose longitudes are different, it is generally thought that one must face east or west, but it is not so because towns of equal latitude are [on a circle] parallel to one of the day-circles, not parallel to [one of the] great circles, whereas the east-west line is parallel to one of the great circles. Therefore, the azimuth of such localities is inclined toward the north from the east or west. The calculation of the azimuth of localities belongs to practical books.

[4] What is needed most is the determination of the azimuth of Mecca. The longitude of Mecca has been given as 77°10′, and its latitude as 21°40′. Then, when the Sun is at either one of these two degrees, 7°20′ Gemini or 22°40′ Cancer, it will transit the zenith of the people of Mecca. If the longitude of any given town is a greater longitude than that of Mecca and the equinoctial moves by the amount of the longitudinal difference from noon, or if the [town's] longitude is less, and the same amount [of equinoctial movement equivalent to the longitudinal difference] will remain until noon, it will [then] be noontime for the people of Mecca, the direction of the shadow will be the direction line, and the direction of the Sun will be the azimuth of Mecca. If the altitude of the Sun at that time is determined by observation, the azimuth of Mecca will be known. As this is sufficient, let us end this Part here—God is the Provider of succor.

BOOK IV
On Determining Distances and Sizes of Bodies, in Six Chapters

CHAPTER ONE
On Determining the Measure of the Earth's Sphere and Its Explication

[1] Since it has become well known that the Earth is spherical, its center is the center of the World, its outer surface is parallel to the surface of the zodiacal orb and the distances in any direction are the same, [and] since any celestial equator is divided into 360, with each division called a degree, [then] an equator can also be imagined on the Earth such that its divisions are according to the divisions of the orb. Therefore, anyone moving beneath the meridian circle, such that the altitude of the pole or the maximum altitude of the Sun or any other planet increases or decreases one degree, that person will have traversed the amount of one degree on the Earth. If this amount is multiplied by 360, the size of the Earth's equator will be known. [Now,] from [the size of] the equator, which is well known to the practitioners of the science of surveying, the sphere's diameter, the area of its outer surface, and size can be determined. [This is] because the product of

the radius multiplied by half [the circumference of] the equator gives the area of the plane of the equator, and this amount is a quarter of the area of the surface of the sphere. The practitioners of the discipline [of surveying] have proven these statements and clarified them.

[2] [Concerning this matter,] Ptolemy, who is the master of this science, undertook testing and observation and found the amount of one degree of [the circumference of] the Earth to be $66+\frac{2}{3}$ miles, each mile [being] 3,000 cubits, each cubit 32 digits, [and] each digit 6 barleycorns laid edge to edge. Therefore, the circumference of the Earth is 24,000 miles. [Then] the diameter of the Earth is 7,636 miles, since the ratio of the circumference to the diameter is as the ratio of 22 to 7, approximately, as Archimedes has proven. The surface area of the Earth is 183,264,000 [square] miles, and one quarter of this amount is the area of the inhabited quarter. If we determine the miles [of the length] of the complement of the obliquity and multiply it by the diameter, the area of the settled portion will be 33,812,208 [square miles], which is approximately $\frac{1}{6}+(\frac{1}{6}\times\frac{1}{10})$ of the Earth['s surface area].

[3] During the reign of the Caliph Maʾmūn, a group of the learned reassessed this matter at his order and found the one-degree portion [of the circumference of the Earth] to be $56+\frac{2}{3}$ miles, each mile [being] 4,000 cubits, each cubit 24 digits, [and] each digit being 6 barleycorns laid edge to edge. This amount is close to the amount of Ptolemy's [unit for a] mile, since the difference that arises from the number of cubits [of each mile] is eliminated due to [the difference in] the number of digits [of each cubit], whereas there is a difference in the number of the miles. Thus, the circumference of the Earth is 20,400 [miles], its diameter 6,491 [miles], [its] surface area 132,416,400 [square miles], the width of the settled [portion] 3,763 [miles], [and] the surface area of the settled [portion] 24,425,633 [square miles], each mile being $\frac{1}{3}$ of a parasang.

CHAPTER TWO
On Determining the Ratio of the Moon's Size to the Earth

[1] When one reflects on lunar eclipses that are latitudinally and directionally equal, but at varying distances from the Earth, [it can be seen that] the higher [i.e., farther away from the Earth] the Moon is during [the eclipse], the less the duration [of the eclipse], and the closer it is to the Earth the longer the duration. [Now,] this closeness and farness in distance can be due only to the epicycle orb, because a lunar eclipse always occurs at the farthest distance of the eccentric orb. This indicates that the farther the Earth's shadow is from the Earth the narrower it is. Thus, it resembles a pinecone in shape, whose base is the Earth; for if [the shadow] were wider the farther away it is, the duration of the eclipse at the [shadow's] apex would be longer, but it is not; and if it were of a uniform width [and of a] cylindrical shape, the duration [of eclipses] at all distances would be the same, but it is not. Since the [Earth's] shadow is narrower the farther it is from the Earth, the Sun is larger than the Earth, for if it were smaller than the Earth, the shadow would be wider the farther it is from the Earth. If the Sun were equal to the Earth, the shadow would be cylindrical. Since the shadow is a cone with the Earth as its base, no circle can be assumed on that cone larger than the equator of the Earth, which is [its] base. Since at the place of the Moon the shadow is smaller than the equator of the Earth and completely obscures the body of the Moon, the Moon is smaller than the Earth. Therefore, based on this consideration, it has become known that the Sun is larger than the Earth, and the Moon is smaller than the Earth. Since the Sun is larger than the Earth, the shadow gets smaller until it reaches a point where it vanishes.

[2] For determining the size of the Moon and the shadow, two lunar eclipses were sought during both of which the Moon would be at

the apex of the epicycle; during one, a quarter of the diameter of the Moon's surface was eclipsed, while during the other, half [of the diameter was eclipsed]. In the first eclipse, the latitude of the Moon was found to be 49 minutes plus a fraction, and in the second eclipse, 41 minutes plus a fraction. Therefore, it was determined that for every 8 minutes minus a fraction that the latitude decreases, one quarter of the size of the Moon's diameter is added in eclipse. One quarter of the Moon's diameter was taken to be 3 digits, since the whole diameter is taken to be 12 digits. Since in the second eclipse half of the diameter was eclipsed, the shadow circle had passed through the center of the Moon; thus, the amount of the latitude of the Moon was [equal to] the radius of the shadow circle, inasmuch as the center of the shadow circle always adheres to the zodiacal equator, facing the center of the body of the Sun. When the Moon's latitude in the second eclipse, which is equal to the radius of the shadow, is multiplied by 3 digits and divided by 8 minutes minus a fraction, the result will be fifteen and a half digits, which is the radius of the shadow at the Moon's epicycle apex—the diameter of the Moon being 12 digits.

[3] Thereafter, two other eclipses at perigee were sought, such that—as previously stated—one quarter of the Moon's diameter would be eclipsed in one, and half in the other. Using the same method mentioned, the radius of the shadow circle at the perigee was determined and found to be $16+\frac{2}{6}$ digits. It was then determined that when the shadow was closer to the Earth by the amount of a diameter of the epicycle orb, the diameter of the shadow increased by $\frac{5}{6}$ of a digit, because between the first two eclipses and the second two eclipses was a difference of not more than the size of the diameter of the epicycle—with no conceivable difference due to the eccentric.

[4] Since the radius of the epicycle of the Moon, as has been stated, is $5+\frac{1}{4}$ degrees—the radius of the inclined [orb] being 60 degrees—and the farthest distance of the eccentric is tangent to the surface of the

inclined [orb], considering circles, not bodies, then [the distance] from the apex of the epicycle orb to the center of the Earth will be $65+\frac{1}{4}$ degrees by this standard [of measurement]. This is the axis of the shadow cone. Since the radius of the epicycle is $5+\frac{1}{4}$ degrees, the diameter will be $10+\frac{1}{2}$ degrees. It has been determined that for every $10+\frac{1}{2}$ degrees that the shadow is closer [to the Earth], its radius increases by $\frac{5}{6}$ of a digit. [Thus] at this distance that the apex has from the Earth, the radius of the shadow increases by [the amount of] 5 digits plus a fraction. When this amount is added to the $15+\frac{1}{2}$ digits that were found when [the Moon was] at the apex of the radius of the epicycle, the result is the radius of the base of the shadow [cone], and this is equal to the radius of the Earth. Therefore, the diameter of the Earth is approximately 41 digits—the diameter of the Moon being 12 digits.

[5] When 41 is divided by 12, the result is $3+(2+\frac{1}{2})\times\frac{1}{6}$. Therefore, the ratio of the diameter of the Moon to the diameter of the Earth is 1 to $3+(2+\frac{1}{2})\times\frac{1}{6}$, [whereas] in Ptolemy's calculation it has been given as $3+\frac{2}{5}$. In Book Twelve of his work, Euclid proved that the ratio of the cube of the diameter of one sphere to the cube of the diameter of another sphere is equal to the ratio of the size of [the first] sphere to the size of that other sphere. If we cube the diameter of the Moon, 1 [multiplied] by 1 by 1, it will still be 1; and if we cube the diameter of the Earth, $3+\frac{2}{5}$ [multiplied] by $3+\frac{2}{5}$ by $3+\frac{2}{5}$, it will be $39+\frac{1}{4}$. This will the ratio of the [size of the] Moon to the Earth, that is, [the size] of the Moon to [the size of] the Earth is as 1 to $39+\frac{1}{4}$, which is what is required. If anyone wants to determine the surface area of the Moon, its diameter and [the size of] its body in parasangs, miles and cubits, it is possible, since these amounts are known for the Earth—God is all-knowing.

CHAPTER THREE

On Determining the Distances of the Moon from the Earth

[1] [For] each amount that is to be determined, there needs to be a scale, such as the yardstick for surveying lands and determining the measurements of fabrics. For determining the [size of] bodies and distances, the practitioners of the discipline have made the Earth the scale; then by [the size of] its body they measure the [size of heavenly] bodies and by its radius the distances. The convention is to take the scale as one unit and measure other amounts in terms of its units.

[2] Since the ratio of the Moon's diameter to the Earth's is known, and the Moon's diameter at its farthest distance is approximately 32′ of the circumference of the inclined [orb], and the ratio of the circumference to the diameter is as the ratio of $3+\frac{1}{7}$ to 1, then the ratio of the Earth['s diameter] to the diameter of the inclined [orb] is known, that being approximately 1 to 60. Therefore, the Moon's farthest distance on the eccentric orb from the Earth's surface is 59 degrees; and its farthest distance at apex and apogee, which is the limit of the Moon's distances, is $64+\frac{1}{4}$ degrees. As the distance between the two centers is 10° 19′—the radius of the inclined [orb] or of the parecliptic being 60 degrees—the distance of the eccentric perigee from the equator of the inclined [orb], i.e., the thickness of the complementary [body], is double this amount, or 20° 38′, and the radius of the epicycle is $5+\frac{1}{4}$ degrees. By subtracting these two amounts from 59, the remainder is 33° 7′, which is the Moon's nearest distance from the Earth—the radius of the Earth being 1 degree. Since the Earth's radius, according to Ptolemy's considerations, is 3,818 miles, which is approximately 1,273 parasangs, the Moon's nearest distance from the Earth's surface is 126,440 miles. Adding the miles of the Earth's radius to this amount, it becomes 130,258 miles, i.e., approximately 43,419 parasangs, and this amount is the radius of the world of generation and corruption. The

Moon's farthest distance from the Earth is 245,306 miles.

[3] If we want to know the amount of the height of the Earth's shadow: since for a distance of 64+$\frac{1}{4}$ parts from the surface of the Earth, the radius of the shadow decreases by 5 digits, and [since] half of the whole base of the shadow is 20+$\frac{1}{2}$ digits, then this amount [of the shadow] comes to a point at 264 parts, this being the maximum distance of the shadow from the Earth. In miles it is 1,007,952 miles; and in parasangs, 335,984 parasangs. At this distance, the Earth's shadow comes to a point. By this calculation, it is known that the shadow terminates at the nearest distance of Venus and expires in the thickness of [Venus's] orb. This is the purport of this chapter—God is all-knowing of the Truth.

CHAPTER FOUR
On Determining the Size and Distances of the Sun

[1] When [the astronomers] observed carefully, [they found that] the Moon's surface at its farthest distance is almost exactly equal to the Sun's surface at its mean distance—and this estimation can be known by the observation of a solar eclipse. When there are two objects equal in sight but different in distance, the ratio of the diameter of one to another is as the ratio of the distance [of one] to the distance [of the other], as indicated by geometrical proof and the rules of the science of optics. The ratio of distance to distance is as the ratio of parallax to parallax counter-proportionally, i.e., the ratio of the Moon's distance to the Sun's distance is as the ratio of the Sun's parallax to the Moon's parallax.

[2] The [astronomers] observed the parallax of the two at this aforementioned distance as precisely as possible and found the Sun's parallax at mean distance to be 1′ 27″ and the Moon's parallax at its farthest distance to be 27′ 10″. Since the ratio of the Moon's diameter to

the Sun's diameter is the same as the ratio of the Sun's parallax to the Moon's parallax, they divided the Sun's parallax by the Moon's parallax, the result being 18+$\frac{4}{5}$. Thus, it became known that the ratio of the Moon's diameter to the Sun's diameter is as the ratio of 1 to 18+$\frac{4}{5}$. It had already been known that the ratio of the Moon to the Earth is the ratio of 1 to 3+$\frac{2}{5}$. Therefore, the ratio of the Earth to the Sun is as the ratio of 3+$\frac{2}{5}$ to 18+$\frac{4}{5}$. If one divides this [latter] amount by the [former], the result will be 5+$\frac{1}{2}$. Therefore, the ratio of the Earth to the Sun is as the ratio of 1 to 5+$\frac{1}{2}$. The cube of 1 is 1; the cube of 5+$\frac{1}{2}$ is 166+$\frac{1}{4}$+$\frac{1}{8}$. Thus, it became known that the Sun is 166+$\frac{1}{4}$+$\frac{1}{8}$ times [as large as] the Earth.

[3] Furthermore, if the ratio of the Sun to the Moon is desired, cubing 18+$\frac{4}{5}$ will be approximately 6,645. So, the Sun is 6,645 times [as large as] the Moon. As for the Sun's distances, since the ratio of the Moon's diameter to the Sun's diameter is equal to the ratio of distance to distance, and the ratio of diameter to diameter is as the ratio of 1 to 18+$\frac{4}{5}$, then the mean distance of the Sun is 18+$\frac{4}{5}$ times the farthest distance of the Moon. Then, we multiplied 64+$\frac{1}{5}$, which is the farthest distance of the Moon, by 18+$\frac{4}{5}$ and obtained 1,208, which is the mean distance of the Sun, the radius of the Earth being 1.

[4] Ptolemy found the distance between the Sun's two centers to be 2+$\frac{1}{2}$ degrees—the radius of the parecliptic at mean distance being 60 degrees—which he multiplied by 18+$\frac{4}{5}$ and obtained 47. If one adds this amount to 1,208, the result will be 1,255, which is the Sun's farthest distance. If one subtracts [47] from this amount [1,208], the remainder will be 1,161, which is the Sun's nearest distance. If one multiplies these amounts by the miles of the Earth's radius, one will obtain: 4,432,698 miles for the amount of the nearest distance; 4,612,144 miles for the mean distance; and 4,791,590 miles for the farthest distance. Therefore, [the distance] from the Earth to the Sun's mean distance is approximately 1,537,381 parasangs—God is all-knowing.

CHAPTER FIVE
On Determining the Distances and Sizes of the Vacillating Planets

[1] For every wandering planet that is below the orb of Mars and that has some perceptible parallax, its parallax at the farthest distance is equal to the parallax of the wandering planet above it at the nearest distance. It therefore became known that the farthest distance of one is contiguous to the nearest distance of the one that is above it, and this same consideration was followed for the upper planets. Let us begin with Mercury.

[2] Mercury: The ratio of its diameter at the farthest distance to its diameter at the nearest distance has been found in theory to be as the ratio of 1 to $2+\frac{1}{3}+\frac{1}{4}$. Since the farthest distance of the Moon, which is the nearest distance of Mercury, has been found to be $64+\frac{1}{4}$, this amount was multiplied by $2+\frac{1}{3}+\frac{1}{4}$, the result being 166, which is Mercury's farthest distance. Its mean distance is the average of these two, i.e., 115—the radius of the Earth being 1. Therefore, in miles the nearest distance is 245,306, the mean distance 439,070 miles, and its farthest distance 633,788 miles.

[3] When Mercury's distances became known, Mercury's diameter at mean distance was found to be $\frac{1}{15}$ of the diameter of the Sun by precise observation and calculation, on the condition that the Sun be also at the mean distance. Since the farthest distance of the Moon is $64+\frac{1}{4}$, and the Sun's mean distance is 1,208, and the ratio of diameter to diameter is as the ratio of distance to distance, we decided to assume for the Earth an amount whose ratio to the distance of the Sun is as the ratio of the Earth's diameter to the Sun's diameter. The ratio of the Moon's diameter to the Earth's is 1 to $3+\frac{2}{5}$. We then multiplied the farthest distance of the Moon by $3+\frac{2}{5}$, the result being 218. Then, the ratio of

218 to 1,208 is as the ratio of the Earth's diameter to the Sun's, and this ratio is taken as the standard measure for all the planets.

[4] Since the ratio of Mercury to the Sun is the ratio of 1 to 15, and the ratio of Mercury's diameter to $\frac{1}{15}$ of the Sun['s] is as the ratio of the Sun's distance to Mercury's distance, therefore the ratio of $\frac{1}{15}$ of Mercury's distance to the Sun's distance is as the ratio of Mercury's diameter to the Sun's diameter. We divide Mercury's distance by 15, resulting in 7+$\frac{2}{3}$. Therefore [the ratio of] Mercury to the Earth is as 7+$\frac{2}{3}$ to 218. We divided 218 by it; one finds that Mercury to the Earth is as 1 to 28 plus a fraction. When both [values] are cubed, one finds that Mercury's size to the Earth's size is as 1 part to 22,000.

[5] Venus: The [proportional] difference between its diameter at the farthest distance and its diameter at the nearest distance in theory was found to be as 1 to7 minus a fraction. When this amount was multiplied by the farthest distance of Mercury—since Mercury's farthest distance is Venus's nearest distance—the result became 1,160. This is Venus's farthest distance, [which is] close to the Sun's nearest distance according to the requirement of the preceding calculation; thus, this is evidence of the correctness of the procedure. The mean distance [of Venus] according to this calculation is 663. Therefore, in miles [its] mean distance is 2,531,335, and [its] farthest distance is 4,428,880 miles.

[6] Venus's diameter at mean distance to the Sun's diameter at mean distance has been found in theory and by observation to be approximately $\frac{1}{10}$. The [measure in] parts of the mean distance, which is 663, was divided by 10, and 66+$\frac{3}{10}$ was obtained; this is the [measure in] parts of Venus—the measure of the Earth being 218. When the [measure in] parts of the Earth was divided by this, the result was 3+$\frac{1}{4}$; it [thus] was found that Venus's diameter to the Earth's diameter is as 1 to 3+$\frac{1}{4}$. Both amounts were cubed; it was [now] found that [the ratio

of] Venus's size to the Earth's size is approximately as 1 to 34+$\frac{2}{3}$.

[7] Mars: The [proportional] difference between its size at the farthest and at nearest distances was found, in theory, as 1 to 7 minus something, like the case for Venus. The farthest distance of the Sun, which is the nearest distance of Mars, was multiplied by this amount; Mars's farthest distance was then 8,764 and the mean distance was 5,008. Thus, the mean distance is 19,120,544 miles and the farthest distance 332,460,952[1] miles. Mars's ratio to the Sun when both are at mean distance has been found to be 1 to 20. The mean distance was divided by 20, resulting in 250+$\frac{2}{5}$. This was divided by 218—the [standard measure in] parts of the Earth—resulting in 1 part and 7 minutes; thus, the Earth's diameter to Mars's diameter is as 1 [part] and 7 minutes. Both were cubed and it was found that the size of the Earth to the size of Mars is as 1 to 1+$\frac{1}{2}$.

[8] Jupiter: The [proportional] difference between its diameter at the farthest and at the nearest distance is as 1 part to 1 [part] and 31 minutes. Mars's farthest distance was multiplied by this amount, the farthest distance of Jupiter becoming 14,168. According to this standard, the mean distance is 11,466—the radius of the Earth being 1. The mean distance is 43,777,188 miles and its farthest distance is 542,093,424[2] miles. By sight, [the ratio of] its [apparent] size to the Sun['s], when both are at mean distance, is half of $\frac{1}{6}$. The mean distance was divided by 12, and the result was 955. [This] was divided by 218, resulting in 4+$\frac{1}{4}$+$\frac{1}{6}$. Thus, the Earth's diameter to Jupiter's diameter is 1 to this amount. Both amounts were cubed; it was [then] found that Jupiter's

1. The correct number should be 33,460,952, but because 332,460,952 is attested by our best witnesses, we have retained it in the text. We do not know the origin of the mistake, whether it is due to Ṭūsī or an early copyist.

2. The correct number should be 54,093,424; again, it is not clear whether the origin of the mistake is due to Ṭūsī or an early copyist.

size is 84+$\frac{1}{4}$+$\frac{1}{8}$ times [the size of] the Earth.

[9] Saturn: The [proportional] difference between its diameter at the farthest and nearest distances is as 1 to 1+$\frac{2}{5}$. When Jupiter's farthest distance is multiplied by this amount, it becomes 19,835, which is Saturn's farthest distance. According to this standard, the mean distance is 17,001. Thus, the mean distance is 44,909,818[1] miles and the farthest distance is 75,730,030 miles. [The ratio of Saturn's apparent size] to the Sun, when both are at mean distance, was found to be as $\frac{1}{2}$ of $\frac{1}{9}$. Saturn's mean distance was divided by 18, resulting in 944+$\frac{1}{2}$. [This] was divided by 218—the [standard measure in] parts of the Earth—resulting in 4+$\frac{1}{3}$. Therefore, the Earth's diameter to Saturn's diameter is 1 to 4+$\frac{1}{3}$. When both amounts are cubed, it will be found that [the ratio of] the Earth's size to Saturn's size is as 1 to 81+$\frac{1}{5}$+$\frac{1}{6}$.

CHAPTER SIX
On Determining the Distance and Sizes of the Fixed Stars

[1] Since all the fixed [stars] had been placed on one orb, their distance was taken to be the same distance, which is equal, according to the aforementioned standard, to Saturn's farthest distance. This [distance] has already been given in terms of the Earth's radius and in miles; in parasangs it is 25,243,343 parasangs. This is the limit of the distance of bodies for which humans have a way to know. When they considered the sizes of the [fixed stars], they classified them into six magnitudes, as has been stated, such that a star of the first magnitude is taken to be, for example, [the size of] a *dirham*, and that of the sixth magnitude to be [the size of] a *dāng* [i.e., one sixth of a *dirham*], according to this standard. This is only an extremely approximate estimation.

1. The correct number should be 64,909,818.

[2] The [apparent diameters of] stars of first magnitude were compared to [that of] the Sun at mean distance. The [ratio] of the most average [first magnitude] star in size to the [Sun's apparent diameter] has been found to be 1 to 20. They [then] divided Saturn's farthest distance by 20, the result being 991+$\frac{3}{4}$. [When] this was divided by 218, 4 parts and 33 minutes was obtained. Therefore, the Earth's diameter to the diameter of the largest stars is as 1 to this amount. When both amounts are cubed, [it is found that] the Earth's size to the size of any of these stars is as 1 to 94+$\frac{1}{5}$. Therefore, the largest of the fixed [stars] is 94+$\frac{1}{5}$ times the Earth. Dividing this amount by 6 gives the difference between each magnitude and another [i.e., the next]; thus, stars of the sixth magnitude are approximately 16 times the Earth, and stars of the fifth magnitude are twice this amount, and so on.

[3] This distance and size that has been calculated for any of the fixed stars is according to [the assumption of] being at the farthest distance of Saturn; if, however, they are farther, both their sizes and distances are greater, and under any assumption they cannot be less than that. From these chapters, it has become known that the smallest of the bodies is Mercury; larger than [Mercury] is the Moon, then Venus, then the Earth, then Mars, then [fixed] stars of the sixth magnitude, [then] the fifth [magnitude], then Saturn, then Jupiter, then stars of the first magnitude, and then the Sun. In conclusion, among the celestial bodies, the greatest is the Sun. These aforementioned distances were based on Ptolemy's reckonings. If desired, they can also be determined based on the reckoning of the moderns according to the preceding [method]—God is all-knowing.

[Ismāʿīlī Ending]

Inasmuch as a general exposition of the science of *hayʾa* [i.e., astronomy; lit., configuration] has been presented, as promised at the beginning of the treatise, we shall end the treatise with this chapter and [this] book. If the blessed temper of the Prince of Iran—may God multiply his glory—were pleased by it, then extreme happiness would come this sincere servant's way; otherwise, his failure is not unusual. If a slip of the pen, infelicity in statement, insufficiency in meaning, or disparity in concept caught the eye of [his] noble [presence], may he cover it under the cloak of his forgiveness, and accept proffering of an excuse; besides incapability, extreme rush and disorientation of mind due to all kind of distractions, this current composition, due to urgency, went on without investigation and contemplation. May God—may He be glorified and exalted—support that house of glory and nobility, and [may He] bestow what is needed for stability and order and what is necessary for the attainment of [this] aspiration—He is kind and the answerer of prayers.

[Revised Ending]

Inasmuch as a general explanation of the science of *hayʾa* [i.e., astronomy; lit., configuration] has been presented, as promised at the beginning of the treatise, we shall end the treatise with this chapter and [this] book. If a slip of the pen, infelicity in expression or concept, or a mistake comes into view, it should be forgiven, for it has been written extemporaneously in a state of haste—God, may He be exalted, is all-knowing of the Truth and to Him is the return and end [of all beings].

[Short Ending]

Inasmuch as a general explanation of the science of *hayʾa* [i.e., astronomy; lit., configuration] has been presented, as promised at the beginning of the treatise, we shall end the treatise with this chapter and [this] book. If a slip of the pen, infelicity in expression or concept, or a mistake comes into view, it should be forgiven, for it has been written extemporaneously in a state of haste. May God—may He be glorified and exalted—bestow what is needed for stability and order and what is necessary for the attainment of [this] aspiration—He is kind and the answerer of prayers.

Ḥall-i Mushkilāt-i Muʿīniyya

Ismāʿīlī Preface

In the Name of God, the Compassionate, the Merciful

[1] When the divine providence pulled back the veil of waiting from the face of the writer of this draft and he was granted the privilege of serving the Prince of Iran, chief and pride of the worldlings, most magnificent chief, pride of the Arabs and Persians, Muʿīn al-Dawla wa-al-Dīn, protector of Islam and Muslims, treasure of kings and sultans, the most noble pedigree in the world, Abū al-Shams—may God magnify his glory and conjoin his morning and evening with auspiciousness—and the time helped him with the attainment of the wish and the accomplishment of the desire, i.e., with granting the fortune and attaining the proximity and benefit of having a conversation with such a generous person and noble man, the kind temper of this unequaled [man] of the time. Even though he is aware of the subtleties of the sciences and the secrets of the inner meanings, nevertheless, in order that this privilege becomes perfect and this nobility whose most sincere of his servants attained continues, he became pleased to command a scientific discourse that had started about the treatise that is named the *Muʿīniyya* and had already been composed for the flourishing library of this prince—may God magnify his glory. During those discussions, as a result of his perfect sharpness and extreme proficiency, a noble order was issued to furnish some explanatory notes where a statement was phrased in an ambiguous way or there was obscurity and

uncertainty in order that when one comes to study [it], [one's] mind does not weary from elucidating those concepts, and memory need not be reminded of an explanation. Thus, in conformity with [what was mentioned in] this introduction and because obeying a commandment is one of the requisites of service and conditions of compliance, these pages have been arranged so that under each designated chapter something of what occurs to this humble one's imagination may be penned, to the extent of [his] ability. God willing, it will be ennobled through the honor of [bringing] satisfaction and will be regarded with contentment. May God—may He be glorified and exalted—make the essence of this honorable [person] the source of virtue and the confluence of glorious deeds—He is the answerer of prayers and kind.

Revised Preface

In the Name of God, the Compassionate, the Merciful

[1] After the composition of the *Mughniyya* treatise on *hayʾa* was completed, one of the grandees at whose request it had been written showed great interest in reading it and entreated that a record of the solutions to the doubts be made, as was promised therein, and an explanation of some passages that were problematic for him be appended. Hence, these pages have been arranged so that under each designated chapter something of what occurs to this humble one's imagination may be penned as required. God willing, it will be acceptable. May the great ones who see it be not constrained to correct the mistakes that are correctable and forgive the disorderliness. And God gives success in every good thing and guides to every truth—He is the answerer of prayers and kind.

The Text of *Ḥall-i Mushkilāt-i Muʿīniyya*

Section [One]
Concerning That When the Colatitude of a Fixed Star is Greater Than the Excess of the Local Latitude over the Obliquity, It Might Become Invisible or Visible after Having Been Either Permanently Visible or Permanently Invisible

[1] In Chapter Three of Book II, it has been stated that there would be a time when a permanently invisible star would become visible, on the condition that its colatitude be greater than the excess of the local latitude over the obliquity. And there would also be a time when a permanently visible [star] would become invisible, also on this condition. To clarify this problem, we say that it has been explained in the same chapter that each of these [fixed] stars has two circuits: one latitudinal circuit about the zodiacal orb's pole that never becomes larger or smaller; and the other a diurnal circuit about the equinoctial pole. This [latter] circuit becomes greater or smaller due to the motion of the star in the sequence of the zodiacal signs and [due to] the increase or decrease of its distance from the equinoctial. Because whenever the distance of the stars from the equinoctial becomes greater, a [given] star will become closer to the equinoctial pole; thus, its circuit will become smaller. Conversely, when the distance becomes less, the circuit will become larger. The largest circuits that are permanently visible are those whose distance from the equinoctial pole is in the amount of the local latitude. Therefore, every star whose distance from the equinoctial is equal to the local colatitude is on this circuit, and every star whose distance from the equinoctial is greater than this is permanently visible. That whose distance is less than this has both visibility and invisibility. The distance [from] the equinoctial will increase or decrease up to the limit whereby the star in longitude reaches the

beginning of Cancer or Capricorn. After that [point], if it has been increasing it will decrease, and if it has been decreasing, increase. Thus, for every star whose distance from the equinoctial is assumed to increase, its maximum increase [in distance] will be when it reaches one of [these] two points. And when it reaches one of these two points and its distance [from the equinoctial] becomes[1] greater than the local colatitude, it will not fall on a permanently visible or permanently invisible circuit. And because the zodiacal orb's pole moves about the equinoctial pole with the primary motion, it will have a circuit, and on its circuit it has an altitude above which it cannot go, which is equal to the sum of the local latitude and the obliquity (*mayl-i aʿẓam*); this is because the altitude of the equinoctial pole is equal to the local latitude and the distance of the zodiacal orb's pole from it is equal to the obliquity. There is an altitude that it cannot be less than, and that is in the amount of the excess of the local latitude over the obliquity; the reason is that since [the distance] from the horizon to the equinoctial pole is equal to the local latitude, and the pole of the zodiacal orb is closer to the horizon by the amount of the obliquity, [then] between the horizon and the zodiacal orb's pole there remains the amount of the excess of the local latitude over the obliquity. Therefore, every star whose distance from the pole of the zodiacal orb, i.e., its colatitude, is this amount when it [i.e., the star] is at the beginning of a solstice will rotate along a [day-] circle that is tangent to the horizon. If its colatitude is less than this amount, it will fall on circuits that are either permanently visible or permanently invisible. And if it is greater than this amount, it will never fall on these circuits. And this is an elucidation of the problem as much as possible.

[2] An example of this is that the latitude of the star Canopus is 75° south; its colatitude is 15°. When it is at the beginning of Cancer, the

1. "becomes" should be "does not become", as it was revised in some copies.

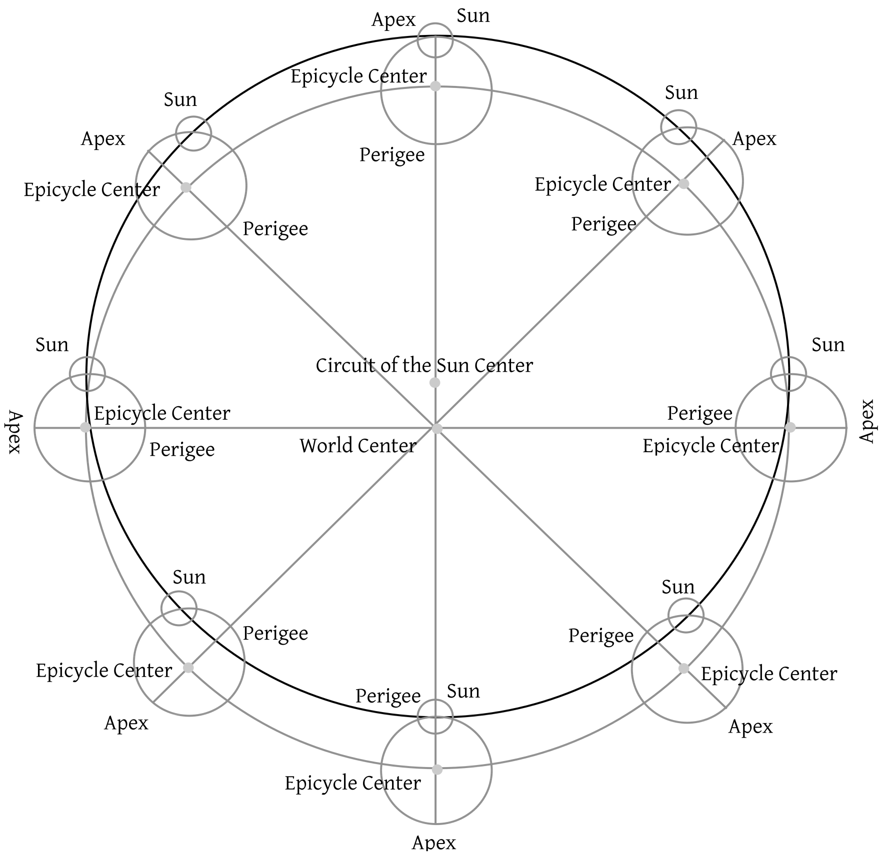
declination of its degree is close to the obliquity. Thus, at the time when [the first of] Cancer is at midheaven in a city whose latitude is 36°, it will be 36° from the horizon to the equinoctial pole below the Earth. At this time, the zodiacal orb's pole that is below the Earth is at its closest position to the horizon, [and the distance] between it and the horizon is in the amount of the excess of the local latitude over the obliquity, approximately 12°. The distance of Canopus from the [ecliptic] pole, i.e. its colatitude, is 15°; thus, it has risen 3° above the horizon. And when it reaches the first of Leo, and it comes to be 3° less in the declination of its degree, its distance from the equinoctial [equator] will become greater by this amount, [so] it will fall on a permanently invisible circuit. Thus, as long as it is in the signs of Gemini and Cancer, it will become visible [and invisible]; in the other ten signs, it will be permanently invisible. One can depict this on a globe.

Section [Two]
On Why the Eccentric Orb Was Chosen for the Sun over the Epicycle

[1] In Chapter Four of Book II, it has been stated that the eccentric orb and the epicyclic orb amount to the same thing in accounting for the variation in the movement of the Sun, and, whichever is posited, the intended result will be obtained. However, the eccentric is more nearly simpler, for the reason that the motion of the Sun on the circumference of the epicycle and the motion of the epicycle on the circumference of the deferent will result in an eccentric circuit for the body of the Sun. Thus, from the positing of an epicycle, there follows the positing of an eccentric, [whereas] from the positing of an eccentric there does not follow the positing of an epicycle. For this reason, Ptolemy posited an eccentric for the Sun. For an explanation of this matter, [let] us conceive the Sun to have a deferent orb whose center

is the center of the World and an epicyclic orb whose center is on the circumference of the deferent moving in the sequence [of the signs]; the Sun moves on the circumference of the epicycle in such a way that in the upper half it moves opposite the motion of the deferent, i.e., in counter-sequence [of the signs], while in the lower half in the sequence [of the signs] like the Moon, and both revolutions are completed altogether, so that when the Sun reaches a given point on the epicycle, the epicycle center will likewise have reached a given point on the deferent, each having completed one full revolution. Then on this assumption, from the motion of the body of the Sun with respect to the center of the World, a circle will result whose center is eccentric to the center of the World. The farthest position on that circle is a point whereby the Sun is on that point at the epicyclic apex; the nearest position is where the Sun is at the epicyclic perigee. And since the movement of the Sun on that side that is farther from the center of the World is in the counter-sequence [of the signs] on the epicycle, the slowness in the motion of the Sun [will result] on the apogean half, and the fastness will result from the motion on the other half, in which the two motions are in the same [direction].

[2] So as to facilitate the conception [of this], we have drawn a figure on which we have indicated in several places the epicycle orb and the body of the Sun according to different states of the motion. And we have drawn the circuit resulting from the motion of the body of the Sun in black, so that when one examines it, this matter will become clear.


Apex
Sun
Epicycle Center
Perigee
Sun
Apex
Epicycle Center
Perigee
Sun
Apex
Epicycle Center
Perigee
Sun
Apex
Epicycle Center
Perigee
Circuit of the Sun Center
World Center
Sun
Perigee
Epicycle Center
Apex
Sun
Perigee
Epicycle Center
Apex
Sun
Perigee
Epicycle Center
Apex
Perigee
Sun
Epicycle Center
Apex

[Figure 1]

Section [Three][1]

Concerning the Solution of the Doubt Arising with Regard to the Motion of the Center of the Lunar Epicycle on the Circumference of the Deferent, and the Uniformity of that Motion about the Center of the World

[1] In the Fifth Chapter of the Second Book, in the midst of [the description of] the configuration of the orbs of the Moon, this doubt occurs. As already pointed out, the same doubt [also] arises in the orbs of other planets, whereby it is assumed that the motion of the epicycle center is on the circumference of the deferent and its uniformity is with respect to the equant center. The upshot of the discussion is that since it is not possible for the motion of the celestial bodies to intensify or weaken, speed up or slow down, reverse direction or turn [from its course], except in relation to us, each sphere that moves must be such that its motion is uniform about its center. If that motion is considered with respect to a point other than its center, it will certainly be non-uniform, just as we have said for the Sun that its motion on the circumference of the eccentric is uniform about its center, but non-uniform about the center of the World. However, it is not reasonable to assume that a motion be non-uniform about its own center and uniform about a point other than its center, while maintaining those principles [regarding the motion of the orbs]. Therefore, anyone who investigates this science and who would posit for every motion an orb causing that motion must take the orbs to be such that

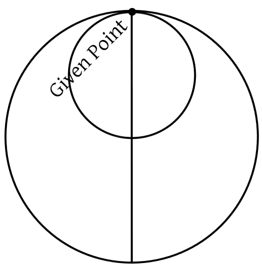
1. For a discussion of this chapter and a comparison with Naṣīr al-Dīn's handling of the Ṭūsī couple in his *Memoir on Astronomy*, see F. Jamil Ragep, "From Tūn to Toruń: The Twists and Turns of the Ṭūsī-Couple," in *Before Copernicus: The Cultures and Contexts of Scientific Learning in the Fifteenth Century,* edited by Rivka Feldhay and F. Jamil Ragep, 161–97 (Montreal: McGill-Queen's University Press, 2017).

the aims of this chapter—namely, the uniformity of motion about the center of the World along with the [continuous] equality of distance from the center of the deferent—be realized and that the motion of the orbs be in actual fact uniform. If [one] adds to or takes away from the number of the orbs, there is no objection against him; however, if he makes alterations in the models (*uṣūl*), which have been found through observation, or overlooks the conformity of some of the principles or premises, he misses the mark. Since Ptolemy, who set forth the principles with dispatch and who was the master of observation, did not take into account [physical] bodies and contented himself with setting forth lines and circles according to his purposes, he, and all those who follow his methods, have freed themselves from an obligation to this commitment. However, a group among the moderns, who have introduced an account of the corporeality of the orbs and a conception of the principles of the motions that they have found by observation, have undertaken this commitment and similar things.

[2] The solution of this doubt, *comme il faut*, is based upon geometrical lemmas. And because in the treatise nothing of that approach [i.e., geometrical reasoning] has been mentioned and [instead] a [summary] account of the problems was deemed sufficient without any geometrical proof, here too, conforming to that [approach], we will write down an indication, in so far as possible, of how to resolve the doubt in such a way that some of [our] intentions may be conceived—God willing. Now we say: in the cited chapter, it was shown that the motion of the epicycle center about the center of the World is uniform. It follows that the mover that gives it this motion has as its center the center of the World. Now the mover of the epicycle orb with this motion is the inclined orb of the Moon or another orb whose center is the center of the inclined orb. It is accepted that the distances of the epicycle center from the center of the World vary, but with respect to another point, such as the deferent center, they are equal. This can be [described]

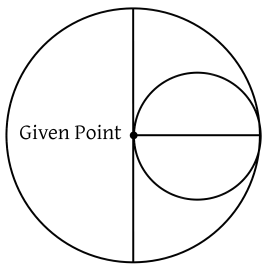
in this way: while the inclined orb gives the center of the epicycle a circular motion, another mover moves it rectilinearly toward the center of the World, such that [the epicycle] approaches the center of the World in one half of the inclined orb's revolution. Afterwards, it causes [the epicycle] to move, also rectilinearly, away from the center of the World in an upward direction, so that when the revolution of the inclined [orb] is completed, the epicycle center will return to its original position, which is the maximum distance from the center of the World. Thus, with this motion, in one half of the revolution, for example, it is closer to the center of the World and in the other half farther away. From the combination of the motion of the inclined orb with this motion, which we have assumed to be rectilinear, an eccentric circuit with respect to the center of the World will result for the epicycle center that is similar to a circle—even though, in fact, it is not a circle—and the uniformity about the center of the World will still be preserved.

[3] The rectilinear motion of the epicycle center away from the circumference of the inclined [orb] in the direction of its center, and afterwards its return on that same bearing until it reaches the circumference, without there resulting in any tearing or mending, or there being a rupture in the path of the circular motion, can be [described] in the way that we are going to mention. Before that, we will set forth a lemma, so that one may be better able to grasp the concept. We say: let us conceive of two circles such that the diameter of one is half that of the other, as in this illustration:



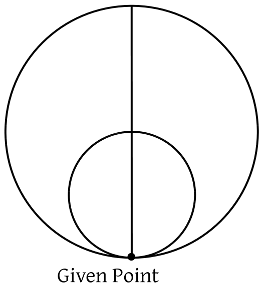
[Figure 1]

[4] At the point of tangency of both circles, let us draw a diameter that passes through both circles. And let us assume the larger circle to move in the counter-sequence [of the signs] and carry the smaller circle, and the smaller circle to move in the sequence [of the signs], the [smaller circle] carrying a given point, which in this illustration coincides with the point of tangency, in such a way that, when the larger circle has completed a rotation, the smaller circle will have completed two rotations. It follows from these two different motions that the given point moves rectilinearly on a diameter of the larger circle, never deviating from that line, such that it moves from this endpoint of the diameter to that [other] endpoint and from the latter endpoint to the former endpoint rectilinearly. For example, when the larger circle describes a quarter-rotation, the smaller circle describes half a rotation, [and] the given point [thereupon] coincides with the center of the larger circle and has traversed one-half the diameter of the larger circle, as in the following illustration:



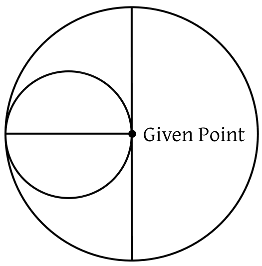
[Figure 2]

[5] Afterwards, when the larger circle has moved another quarter and the smaller circle a half, the diameter of the smaller circle will be coincident with the diameter of the larger circle, and the given point will coincide with the point of tangency, it having traversed the entire diameter, as in this illustration:



[Figure 3]

[6] Afterwards, the smaller circle comes to be in the other half of the large circle, and the given point returns rectilinearly, until the large circle describes a quarter [rotation] and the smaller circle a half, when once again, the given point coincides with the center of the large circle, having traversed half of the diameter, as in this illustration:

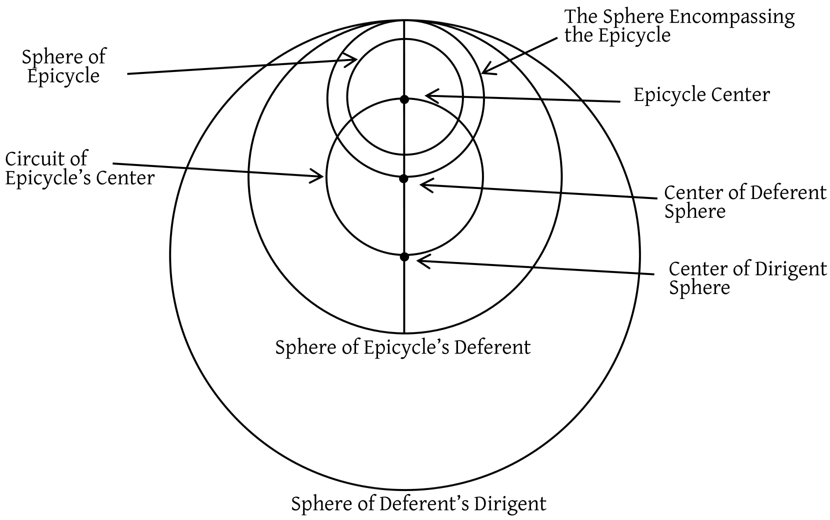


[Figure 4]

[7] Then when the large circle has described another quarter [rotation] and the small circle a half, the given point will reach the beginning endpoint of the diameter and comes to its own [initial] position. Thus, with one rotation of the large circle and with two rotations of the small circle, this point will have twice traversed the length of the diameter of the large circle rectilinearly: once from the first endpoint to the second endpoint and another time from the second endpoint to the first endpoint.

[8] If this lemma is understood, one may easily conceive for the lunar epicycle center a similar motion by means of bodies. This is such

that, in addition to the epicycle orb, we assume three spheres enclosing one another. The first is a sphere that encloses the epicycle and whose center is the epicycle center; we call it the enclosing sphere of the epicycle. Whatever thickness we assume for this orb will be suitable, since no defined limit is necessary for [the thickness]. The second is a sphere that encompasses the enclosing sphere and [it] has a center different from that of the latter, such that the circumferences of the two orbs touch at a single point; we call this the deferent sphere of the epicycle. The third is a sphere that encompasses the deferent sphere, just as the deferent encompasses the enclosing [orb], in such a way that the three spheres are tangent at that one point. And the radius of [this] dirigent sphere is equal to the sum of the eccentricity that we stated in the chapter on the Moon, i.e., 10;19, plus the radius of the epicycle, i.e., 5;15,[1] plus the amount of the thickness of the enclosing [orb] of the epicycle. The radius of the deferent is equal to half the eccentricity plus the radius of the epicycle plus the amount of the thickness of the enclosing [orb], as in this illustration:



[Figure 5]

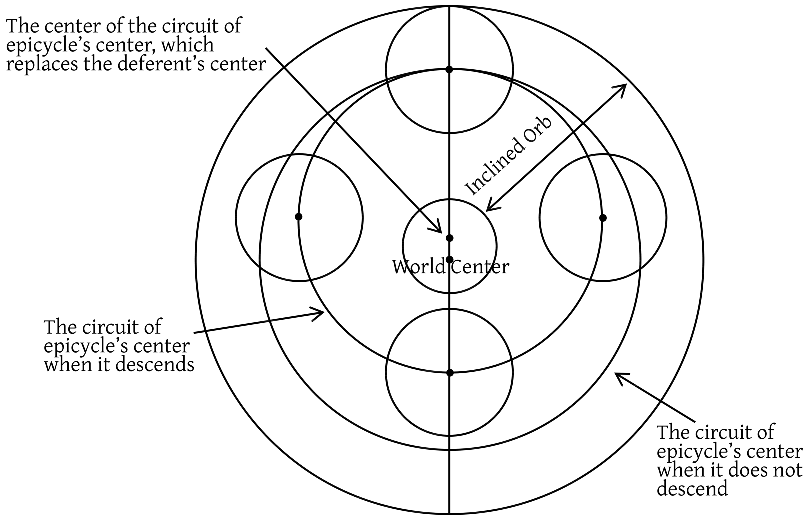
1. Edition should be corrected to read ه يه.

[9] So the circuit of the epicycle center that results from the motion of the deferent passes through the center of the dirigent sphere, and its radius is equal to [half][1] the eccentricity, as has been outlined in black [in Figure 5]. Thus, as we have said, from the motion of the dirigent in one direction and the motion of the deferent in the opposite direction with twice that motion, it follows that the center of the epicycle will descend rectilinearly on the diameter of the dirigent in the amount of twice the diameter of its circuit. And since we have assumed the diameter of the circuit to be equal to the eccentricity, the rectilinear descent of the epicycle center will be in the amount of twice the eccentricity. Afterwards, also rectilinearly, it will ascend until reaching its original position, except that since the apex and [epicyclic] perigee are always aligned with the deferent center, it follows that the epicyclic apex[2] will deviate from the alignment with the dirigent diameter. After half a rotation of the dirigent, the epicycle diameter will be reversed, the apex at the bottom, the perigee at the top. Thus, we assume the epicycle's enclosing sphere to have a motion equal to the motion of the dirigent and in the same direction, so that the apex and the perigee will be brought back by the same amount it has deviated from its alignment with the dirigent diameter to its original position, thus always remaining coincident with the dirigent diameter. Thus, it will descend and ascend rectilinearly along the diameter. This being said, if we conceive this larger orb, namely the dirigent orb, to be embedded in the thickness of the inclined orb of the Moon, just as the epicycle is in the thickness of the deferent, and [if] the inclined [orb] moves uniformly about its own center, so that its rotation is completed with the rotation of the dirigent, there results from the circuit of the epicycle center a figure resembling a circle whose center is removed

1. Only MS K has half.

2. Ignoring the و in the edition.

from the center of the World by the amount of the eccentricity. As the inclined orb carries the epicycle with a circular [motion], while it gradually descends and approaches the center of the World until one half of the rotation of the inclined orb has been completed, the epicycle will have reached its maximum descent in the amount of twice the eccentricity, which is the thickness of the complementary [orb]. It will then be at the perigee, which is facing the original position, namely the apogee. It will ascend, again gradually, also on a circular figure, until reaching the original position. The epicycle center is always on the circumference of a nearly circular circuit, which is taken to replace the [Ptolemaic] eccentric of the Moon. Although the motion of the epicycle center is on the circumference of this [pseudo-] circle, it will [nevertheless] be uniform about the center of the inclined [orb]. One may conceive this concept from the following illustration:



[Figure 6]

The black line is the extent of the descent and ascent of the epicycle center; its maximum is in the amount of twice the eccentricity.[1]

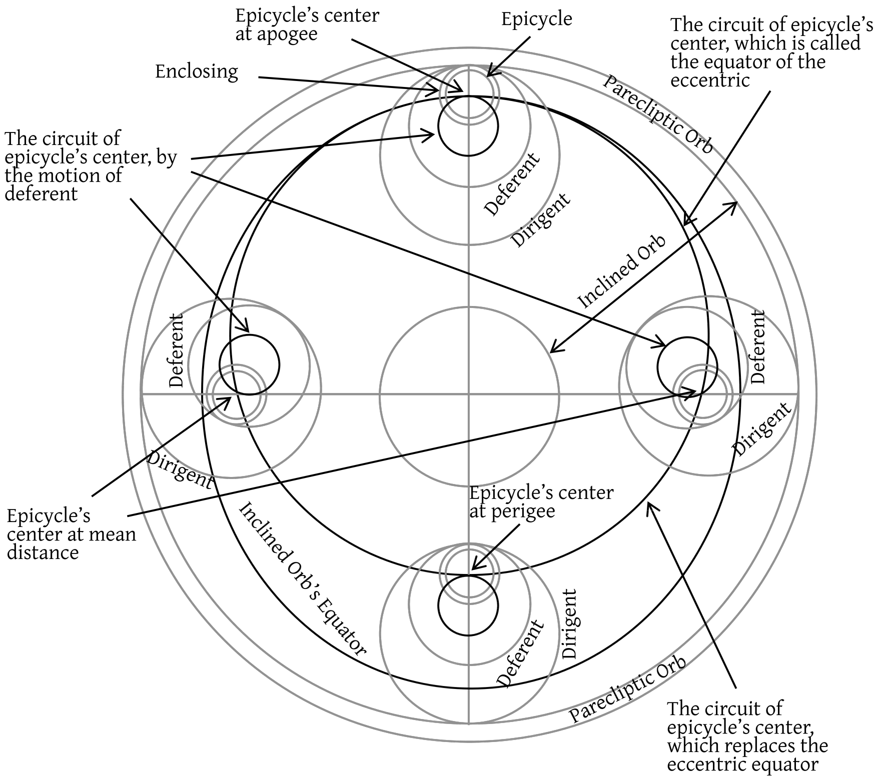
1. This additional sentence is in some of the manuscripts but not in our critical

[10] Now that this introduction has been laid down, six solid orbs for the Moon will be necessary: the first is the parecliptic whose motion is with the motion of the nodes in the counter-sequence [of the signs]; the second is the inclined orb whose motion is equal to the mean motion of Moon in the sequence [of the signs]; the third is the dirigent orb whose motion is equal to that of the Moon's motion of double elongation, which may be designated in whichever direction. Due to this motion, the lunar epicycle is at the apogee at conjunction and opposition, i.e., it touches the outer surface of the inclined orb, and is at the perigee at the two quadratures, i.e., it touches the concavity of the inclined orb; the fourth is the deferent orb of the epicycle whose motion is twice the motion of the dirigent and in the opposite direction; the fifth is the enclosing orb of the epicycle whose motion is in the same direction as the dirigent's motion and equal to it; the sixth is the motion of the epicycle orb with [the Moon's] proper motion, which in the upper half is in the counter-sequence [of the signs] and in the lower half in the sequence [of the signs], and the Moon is moved with this motion. The illustration of these orbs in relation to one another has been set down on another page.

[11] The doubt that has been brought up in [the case of] other planets may also be resolved in this way if the equant is taken to replace the inclined orb and the deferent to replace the eccentric. These orbs, motions, and bodies are not used by geometricians, who [instead] posit motions and explain the anomalies with lines and circles; rather, what Ptolemy has set forth regarding this matter is sufficient [for them]. However, anyone wishing to conceptualize how the motions are in accordance with observations while preserving philosophical principles needs to posit these orbs. This is the exposition of an answer to this difficulty to the best of [our] abilities on this occasion. However, with-

edition; for details, see the variants to this passage.

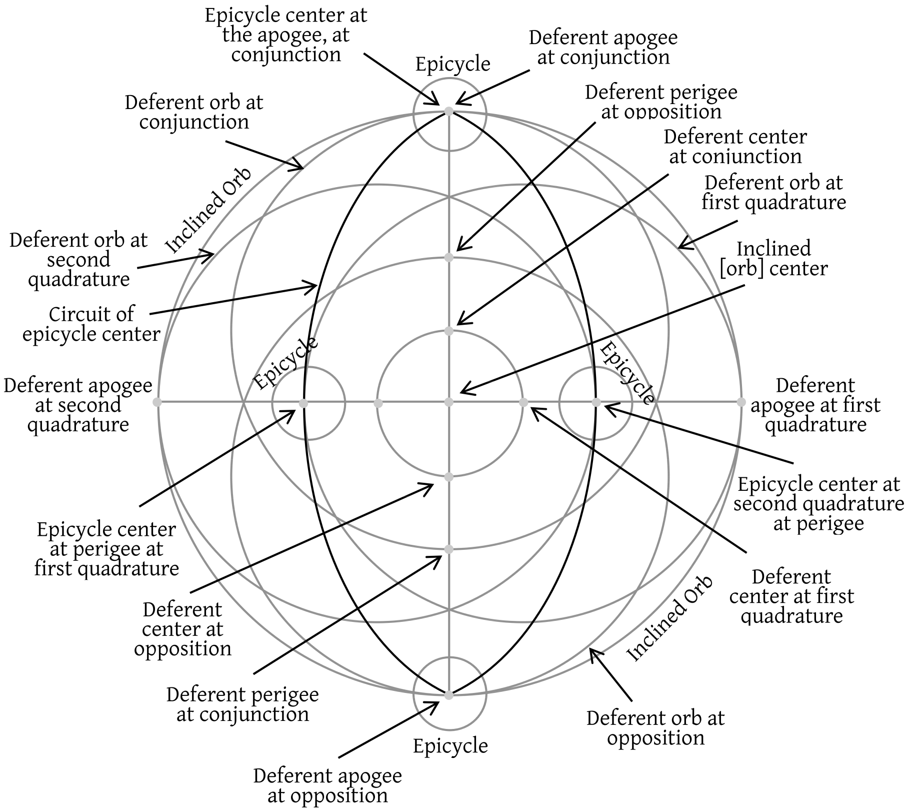
out presenting geometrical theorems, it is not possible to prove that the above motion is strictly rectilinear, that the path of the epicycle center is not a true circle but rather a quasi-circle, and that the deviation from its circularity does not produce a noticeable deviation in the positions of the Moon. Thus, in this place, we must limit [the discussion] to this extent.



[Figure 7]

Section [Four]
An Explanation of the Circuit of the Lunar Epicyclic Center and the Manner in which the Circuit of the Center of the Lunar Epicyclic Orb Comes about Due to the Motions of the Inclined and Deferent [Orbs], Which Was Referred to at the End of the Chapter on the Configuration of the Moon

[1] [This] will be known by examining [the following] figure, as the deferent and epicyclic orbs have been drawn in each of the four positions vis-à-vis the Sun, i.e., conjunction, opposition, and the two quadratures. The circuit of the epicycle center is outlined in black. This figure is [drawn] in this manner, on the assumption that the Sun does not move. However, to be clear, since the Sun does move, the epicycle will describe more than a half revolution from conjunction to opposition and similarly from opposition to conjunction. For Mercury, if in place of the inclined [orb] we put the dirigent orb, one may conceive the exact same figure.



Epicycle center at the apogee, at conjunction
Epicycle
Deferent apogee at conjunction
Deferent orb at conjunction
Deferent perigee at opposition
Deferent center at conjunction
Inclined Orb
Deferent orb at first quadrature
Deferent orb at second quadrature
Inclined [orb] center
Circuit of epicycle center
Epicycle
Epicycle
Deferent apogee at second quadrature
Deferent apogee at first quadrature
Epicycle center at second quadrature at perigee
Epicycle center at perigee at first quadrature
Deferent center at first quadrature
Deferent center at opposition
Inclined Orb
Deferent perigee at conjunction
Epicycle
Deferent orb at opposition
Deferent apogee at opposition

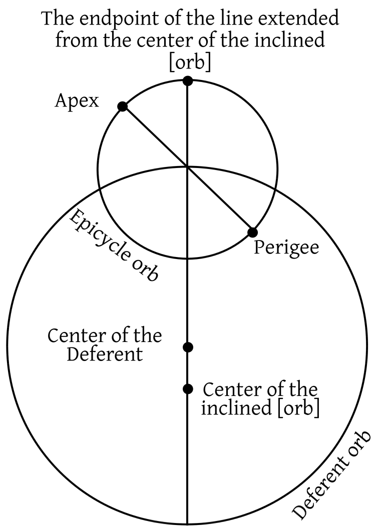
[Figure 1]

Section [Five][1]
On the Configuration of the Epicycle Orbs of the Planets According to the Theory of Abū ʿAlī ibn al-Haytham

[1] This man was a prominent mathematician, and the configuration of the orbs as solid bodies is mostly taken from his statements. He has a treatise explaining the orbs of the planets' epicycles in such a way that the various motions result from them. He states that each of the upper planets has three epicycle orbs, one enclosing another.

[2] The first orb, which is inside the two other orbs, is a [complete] solid orb on one side of which is the planet. That sphere moves with the proper motion of the planet, whose equator we conceive to be outside of the plane of the eccentric equator, intersecting the latter at the two mean distances. Then the diameter passing through the two mean distances is in the plane of the eccentric equator; [as for] the diameter passing through the [epicyclic] apex and perigee, one half [of it] will be in one direction and the other half [of it] in the other direction. If a line is drawn from the center of the inclined [orb] to the center of the epicycle and extended until it reaches the epicycle, then it necessarily intersects the diameter passing through the apex and the perigee at the center of the epicycle. Since this line is in the plane of the eccentric orb, the distance between this line and the diameter of the epicycle [passing] through the apex and perigee is equal to the inclination of the apex and the perigee, according to this illustration:

1. On this section, see F. Jamil Ragep, "Ibn al-Haytham and Eudoxus: The Revival of Homocentric Modeling in Islam," in *Studies in the History of the Exact Sciences in Honour of David Pingree,* edited by Charles Burnett, Jan P. Hogendijk, Kim Plofker, and Michio Yano (Leiden: E. J. Brill, 2004), 786–809.



[Figure 1]

[3] Then we conceive a second orb such that it encloses this [first] orb, and each of these two orbs shares the same center. This orb moves about two poles on the line that comes from the center of the inclined orb, with a motion like that of the epicycle center around the equant center. When this orb moves and carries the first orb with it, then necessarily [both] the apex and the perigee describe two circuits each of whose centers is on the line that comes from the center of the inclined [orb]. These are two small circles each of whose planes is perpendicular to the plane of the eccentric orb, like an [archery] target whose diameter is mounted on the circumference of a shield. So, two positions on [each] circle will be in the plane of the deferent orb, and, when the apex and perigee move along the circumference of [each] circle, they will be in the plane of the eccentric orb whenever they reach these two points. At the midpoint between these two points, they will be at the maximum inclination from the plane of the eccentric.

[4] There follows, however, from this motion a distortion, namely that since the entire epicycle moves with this motion, the diameter passing through the two mean distances goes out of the plane of the defer-

ent and makes a [complete] revolution during which the eastern half of the epicycle becomes western, and the western half becomes eastern. Then in order to rectify this distortion, we conceive another orb, which is the third orb enclosing these [previous] two orbs, in such a way that its center is the center of both orbs. Its two poles are at the two endpoints of the diameter of the epicycle orb that passes through the apex and perigee. Its motion is in the opposite direction of the second orb's motion but equal to it, so that by however much the equator of the epicyclic orb is displaced from its proper place by the motion of the second orb, this orb brings it back to its proper place, and the diameter of the mean distance always remains in the plane of the eccentric orb. However, the apex and the perigee continue to revolve on the above-mentioned circuits, since the poles of this orb are at the two endpoints of the diameter of the epicycle orb and the poles of the second orb are different from these two poles; the distance between each two of these four poles is equal to the radius of the circuit of the apex or of the perigee. Therefore, from these motions it follows that the apex half is always in one direction and the perigee half is in another direction opposite that of the first. In every revolution, the equator of the epicycle arrives twice at the plane of the eccentric equator and [then] passes through it in such a way that the directions interchange.

[5] As the epicycle center traverses the circumference of the inclined [orb] with a motion that is nonuniform with respect to the inclined center and uniform with respect to the equant center, then in the two quadrants that fall in the apogean half it is slower, while in the other two quadrants it is faster. Similarly, the apex and the perigee traverse [their] two circuits with a motion that is nonuniform with respect to the circuit's center but uniform with respect to a point other than the circuit's center that is within the circuit in a position having the status of the equant center, so that the movement of the apex on the circumference of this circuit is slower in two quadrants and faster in [the

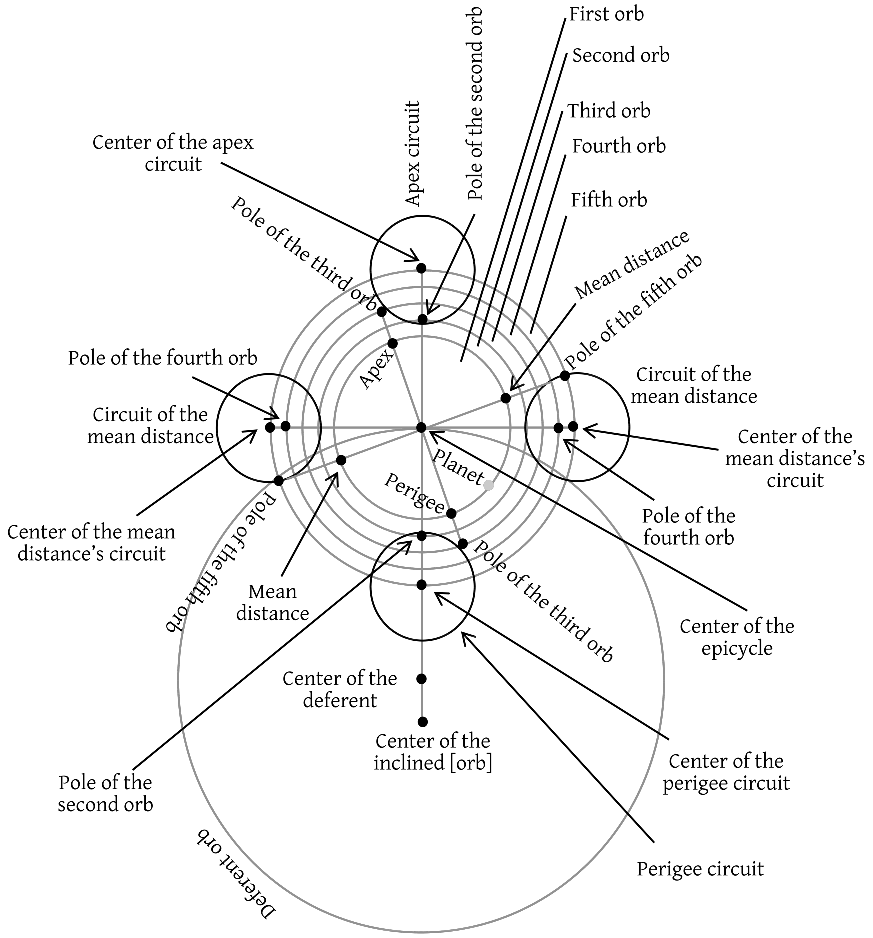
other] two quadrants. And likewise the movement of the perigee, so that the conformity with the motion of center is preserved.

[6] These two small circles are those that the author of *Muntahá al-idrāk* introduced when positing the bodies that are the principles of the motions, and he limited himself to that. Even though these two circles were first posited by Ptolemy in the *Almagest*, nevertheless since Ptolemy limited himself in all cases to circles, this case is consistent with other cases. [On the other hand,] someone who in other places posits bodies but here limits himself to circles is not observing the condition of consistency.

[7] For the two lower planets, to account for the inclination of the apex and the perigee, [Ibn al-Haytham] posited these same two orbs in addition to the epicycle orb; to account for the motion of the slant, he posited two other orbs. The first orb, which is the fourth orb for the epicycles of these [two planets], encloses the other three orbs. The two poles of this orb are two points on a line passing through the epicycle center in the plane of the deferent orb and intersecting at right angles the line that comes from the center of the inclined [orb]. When [this] orb moves, the diameter, which passes through the two mean distances, must necessarily move around these two poles. Thus, there results the motion of the slant, except that since the entire epicyclic equator moves, the apex and the perigee will become displaced from their proper places; the apex goes to the perigee's place and the perigee to the place of the apex. Therefore, a fifth orb encloses these four orbs; its two poles are the two endpoints of the line that passes through the two mean distances. Its motion is opposite and equal to the motion of the fourth orb, so that whatever is displaced from its proper place will return to its original position. Two circuits for the two mean distances result from the motion of the fourth orb, and these are the two small circles that intersect the plane of the deferent orb at right angles, like an [archery] target mounted on a shield,

such that each of the two circumferences are tangent at a point, and one plane intersects the other at right angles. The motion of these two [mean] distances on the circumference of these two circles varies, being faster in one half, slower in the other half, like the motion of center on the inclined orb. When the apex on its own circuit is at the maximum inclination from the inclined plane, the mean distance is in the inclined plane. And when the mean distance is at the maximum inclination from the inclined plane, the apex is in the inclined plane, such that these two latitudes follow in an alternate way. The illustration of the orbs of the epicycles of these two planets, to the extent they can be drawn in a plane, is as follows.

[8] The illustration of the orbs of the upper planets may also be known from this [illustration], if [we] only consider three orbs. This is the exposition of this treatise [of Ibn al-Haytham]—God is all-knowing.



First orb
Second orb
Third orb
Fourth orb
Fifth orb
Pole of the second orb
Apex circuit
Center of the apex circuit
Pole of the third orb
Mean distance
Pole of the fifth orb
Apex
Pole of the fourth orb
Circuit of the mean distance
Circuit of the mean distance
Center of the mean distance's circuit
Planet
Perigee
Pole of the fourth orb
Center of the mean distance's circuit
Pole of the fifth orb
Mean distance
Pole of the third orb
Center of the epicycle
Center of the deferent
Center of the inclined [orb]
Pole of the second orb
Center of the perigee circuit
Perigee circuit
Deferent orb

[Figure 2]

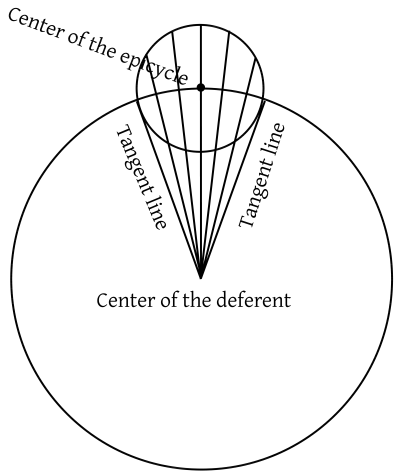
Section [Six]
On Explaining How to Determine the Stationary Positions of the Planets on the Epicycle Orb

[1] In Chapter Ten of Book I[1], we have stated that Ptolemy in the *Almagest* has shown that for a line coming from the center of the World to the epicycle orb and passing through it, if half [the line] inside the epicycle to that outside it between the center of the deferent[2] and the circumference of the epicycle has the same ratio as that of the motion of center to the proper motion of the planet, then when [the planet] reaches that line in the opposite half, [the planet] will become stationary. To explain this, let us draw an illustration of the deferent orb and epicyclic orb, conceiving lines coming from the deferent center to the epicyclic orb. Without too much thought, it is evident that the line tangent to the epicycle does not enter inside the epicycle, and the longest line falling within the epicycle is one that passes through its center, i.e., its diameter. And when a line comes from the center of the World to the epicycle center, that part of the line inside the epicycle orb will be the epicycle diameter, and that which falls between the circumference of the epicycle and the center of the deferent will be the shortest line that comes from the deferent center to the circumference of the epicycle. And of the lines that fall between the tangent line and this line, those falling nearer the epicycle center have longer internal portions and shorter external ones. And those closer to the tangent line will be the converse of this, their internal portions being shorter and their external portions longer. For any two lines assumed

1. This should be Book II.

2. In the revised version, "the center of the deferent" has been correctly changed to "the center of the World." Note that in the text of the edition, *ʿālam* should be *ḥāmil* to reflect the original version.

at equal distance on either side, these portions on both will be equal, as can be clearly seen in this illustration:



[Figure 1]

[2] And it is evident that if one makes the ratio of one quantity to another, e.g., 1 to 10, which is a tenth of it, and then if one makes a ratio from a larger quantity than the first to a smaller quantity than the second, such as 2 to 8, 2 being larger than 1 and 8 being smaller than 10, this ratio being $\frac{1}{4}$, the first ratio is necessarily smaller than the second, $\frac{1}{10}$ being smaller than $\frac{1}{4}$. Therefore, the ratio of half the internal portion of the line closer to the tangent point to the external portion is smaller than the ratio of the internal portion to the external portion of another line farther from that point. And of all ratios, the largest is the ratio of the radius of the epicycle to that line that lies between the center of the deferent and the circumference of the epicycle, since its internal portion is the largest of all internal portions and its external portion is the smallest of all the external portions. And if we consider these lines coming from the center of the World, it will make no difference in the ratios.

[3] Now that these preliminaries have been made clear, we say that any epicycle whose ratio of its radius to the line falling between it and

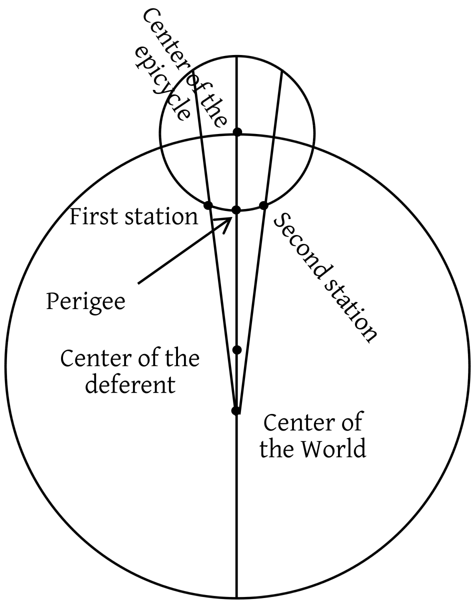
the center of the World is less than the ratio of its motion of center to the motion of the revolving planet, for that epicycle no line can be assumed such that the ratio of half its internal [portion] to its external portion be that ratio, because all ratios are less than the first ratio and the ratio of the first is less than the ratio of the motion of center to the proper motion. Accordingly, all ratios will be less than the ratio of the motion of center to the proper motion. An example of this is the lunar epicycle. When it is at the nearest distance, its radius is found to be 7+$\frac{2}{3}$ parts, and from the Moon's nearest distance to the Earth is approximately 33 parts. When we subtract 7+$\frac{2}{3}$ parts from this amount, there remain 25+$\frac{1}{3}$ parts; this is the length of a line from the center of the World reaching the epicycle. The ratio of the epicycle radius to this amount is approximately $\frac{3}{10}$. The motion of its center is 13 degrees, 11 minutes, and the proper motion is 13 degrees, 4 minutes. The ratio of one to the other is approximately a ratio of equality, and $\frac{3}{10}$ is less than this ratio. Thus, in the lunar epicycle no [such] line can be assumed for this ratio; for this reason, there would be neither station nor retrogression for the Moon.

[4] If we assume that the ratio of the radius of an epicycle orb to the external line is like the ratio of motion to motion in all circumstances, the other ratios, which are less than the ratio of the epicycle radius, are less than the ratio of motion to motion. Therefore, the planet will become stationary at the perigee point of that epicycle and then proceed again with direct motion; for this epicycle there will be no retrogradation. When the ratio of the epicycle radius to the external line is greater than the ratio of the motion, [then] on either side of the radius are found two lines [each of] whose ratios are equal to the ratio of the motion[s]. It follows that every given line between those two lines will have a ratio greater than the ratio of the motion[s], inasmuch as they are closer to the radius. Every line falling outside these two lines on account of being closer to the tangent point will have a ratio smaller

than the ratio of the motion[s]. Therefore, as long as the planet has not reached one of these two lines, it will have direct motion. When it reaches the first line, [the planet] will become stationary, since the ratio of that line is equal to the ratio of the motion. It will then cross those two lines.[1] As long as it has not yet reached the second line on the other side of the perigee, it will retrograde. When it reaches the second line, the [planet] will become stationary. When it passes it and the ratio becomes less [than the ratio of the motions], it will undergo direct motion.

[5] As an example, for the planet Saturn let us assume the epicycle center to be at the apogee of the deferent. Since we take the radius of the deferent to be 60, the eccentricity 3 parts, 25 minutes, [the distance] from the epicycle center to the center of the World 63 parts, 25 minutes, and the radius of the epicycle $6+\frac{1}{2}$ parts, then between the center of the World and the epicycle orb is about 57 parts. The ratio of the [epicycle] radius to this quantity is approximately a ratio of $\frac{1}{9}$. The center moves 2 minutes every day, while the proper motion is 57 minutes. The ratio of 2 to 57 is approximately $\frac{1}{3}$ of $\frac{1}{9}$. Thus, the ratio of line to line is much greater than the ratio of motion to motion. This being the case, two lines will fall on the two sides of the center, one passing through the first stationary point and the other passing through the second stationary point; this is because the ratio of the internal portion of those two lines to the external portion is equal to the ratio of [one] motion to the [other] motion. Between those two lines, the planet will retrograde.

1. In the revised version, "those two lines" have been correctly changed to "that line."



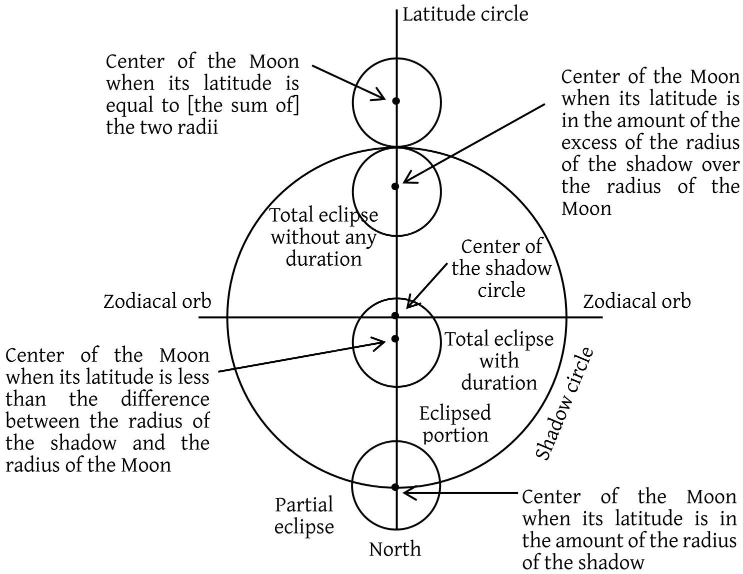
[Figure 2]

Section [Seven]
On Clarifying the Different Circumstances of Lunar and Solar Eclipses with Respect to Differences in Latitude, etc.

[1] In Chapter Thirteen, it was stated that if the lunar latitude is in the amount of the radius of the shadow [plus] the radius of the Moon, the Moon in its transit will become tangent to the shadow circle, and no lunar eclipse will occur. If the [latitude] is greater, the Moon will not become tangent; if it is less [than the shadow radius plus the lunar radius], but greater than the excess of the shadow radius over the lunar radius, there will be a partial lunar eclipse. If it is equal to that excess, there will be a total lunar eclipse without any duration. If it is less than the excess, the lunar eclipse will be total, and there will be duration.

[2] As an example, let us assume a lunar eclipse, where the shadow diameter is 84 minutes and the Moon's diameter is 32 minutes. [Then] the radius of the shadow is 42 minutes and the Moon's radius is 16

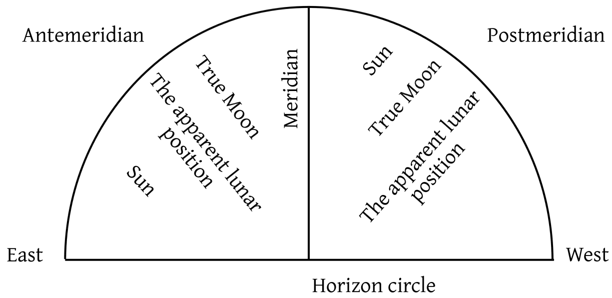
minutes; the sum of these two is 58 minutes. Therefore, if the Moon's latitude is 58 minutes, no lunar eclipse will occur, and the Moon will be externally tangent to the shadow circle. If it is greater, it will not become tangent. If the Moon's latitude is less than this, there will be a lunar eclipse. Since the excess of the shadow's radius over the Moon's radius is 26 minutes, if the Moon's latitude is greater than that, then a partial lunar eclipse will occur. If it [equals] the same amount, a [total] lunar eclipse will occur without any duration. If [the latitude] is less, there will be duration. From this illustration, this matter will easily be understood:



[Figure 1]

[3] Concerning solar eclipses: what we have stated, namely that the times of true conjunction are always closer to noontime than the [apparent] solar eclipse, is due to the fact that since the Moon moves in the sequence of the signs from west to east and the apparent position of the Moon is nearer to the horizon than the true position, then before noon the apparent Moon will reach the Sun first, then afterward the true Moon, and conversely after noon. This meaning should become

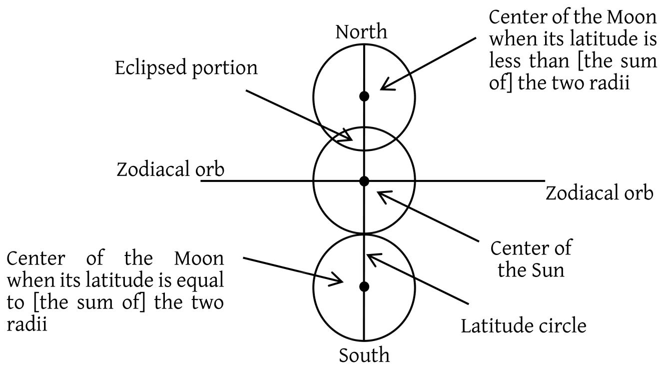
clear from this illustration:



[Figure 2]

[4] From this illustration, in the [part of the] illustration that we have drawn [for] before noon, it is clear that when the Moon is in conjunction with the Sun, [having moved] from west to the east, which conforms to the sequence of the signs, first the apparent position of the Moon will reach the Sun and a solar eclipse will occur. After that the true position will reach it, and there will be conjunction. Thus, the times of the solar eclipse will be sooner than the times of conjunction, and the conjunction will be nearer midday. And in the illustration that we have drawn for after noon, the [Moon's] true position will reach the Sun first, and a conjunction will occur; afterwards, the apparent position of the Moon will reach the Sun, and there will be a solar eclipse. Thus, the times of conjunction will be sooner than the times of solar eclipse, and conjunction will be closer to midday—God is all-knowing.

[5] Let us assume, as an example of a solar eclipse, just as we cited for the Moon, a time when the diameter of the Sun is 32 minutes, and the diameter of the Moon is 34 minutes; thus the [sum of the] two radii is 33 minutes. If the apparent latitude of the Moon is this amount, the Moon will be seen to be tangent to the Sun, and a solar eclipse will not occur. If it is less [than this amount], a solar eclipse will occur. If greater, [the Moon] will not come into tangency, as in this illustration:



[Figure 3]

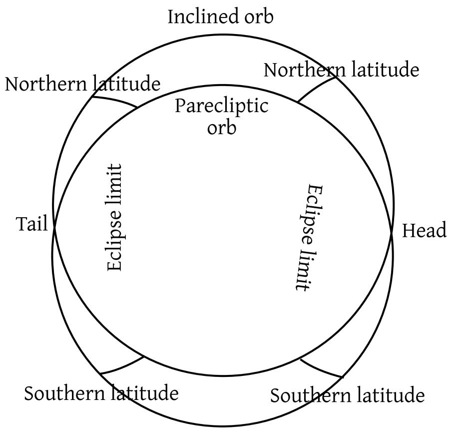
[6] To explain the times between lunar eclipses and solar eclipses, we say: since parallax is not taken into account for a lunar eclipse; and the maximum size of the shadow radius at the time when the Moon is in the epicyclic perigee is up to 46 minutes; and the maximum size of the Moon's radius is up to 18 minutes, the sum of these two amounts being 64 minutes, then when the lunar latitude is more than this amount, a lunar eclipse is not possible. This amount of latitude will occur at a distance of 12 degrees and a fraction from the node, because one degree of latitude will occur at a distance of approximately $15+\frac{1}{2}$[1] degrees. Thus, the Moon's inclined orb may be divided into four parts: in two parts of which a lunar eclipse is possible and in two parts of which a lunar eclipse is not possible. Each of those two parts [in which a lunar eclipse is possible] has a lunar eclipse limit of 24 degrees plus a fraction; each of the other two parts is 156 degrees.

[7] Since a lunar eclipse is at the time of opposition, there is no doubt that the Sun must be in the lunar eclipse limit so that the Moon can be eclipsed when in opposition to it. And since the lunar eclipse limit is no more than 25 degrees, and the Sun traverses this amount in about 25 days, and during this period the Moon is not able to make a complete revolution, hence lunar eclipses are not possible in two consecutive

1. In the revised version, "$15+\frac{1}{2}$" has been correctly changed to "$11+\frac{1}{2}$."

months. In six months, during which the Sun travels from the vicinity of one node to the vicinity of the other node, a lunar eclipse may be able to occur.

[8] If the Sun on rare occasions be at the end of an eclipse limit, for example if it has passed 10 degrees beyond the head and a lunar eclipse occurs, 5 months later the Sun will have traversed about 150 degrees, and the distance between it and the position of the head will be 160 degrees. The tail in these 5 months will have traversed about 8 degrees in the counter-sequence [of the signs]; thus, the distance between it and the Sun is no more than 12 degrees. For this reason, the Sun will have reached the beginning of the lunar eclipse limit; therefore, it will be possible for the Moon to be eclipsed once again five months after the first eclipse, but there will not be a total eclipse for either one.

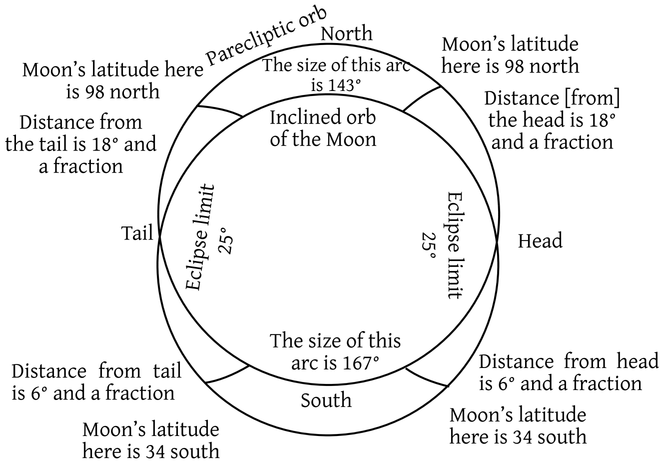


[Figure 4]

[9] But in seven months the Sun will have traversed approximately 205 degrees. If we assume that it was at the beginning of the lunar eclipse limit and the Moon was eclipsed, after seven months the Sun will have passed each of the two nodes and will be 13 degrees beyond the position of the second node. [This is] because if it traverses 12 degrees, it will reach the first node, and [then] if it traverses 180 degrees, it will reach the position of the second node; and when it completes 205

degrees, it will have traversed another 13 degrees. Thus, it has gone beyond the lunar eclipse limit. [This is] despite the fact that in this [same] period, the node has moved 11 degrees in the counter-sequence of the signs, the distance of the node from the Sun thus becoming far too much. Hence, it is not possible for two lunar eclipses to be seven months apart.

[10] As for a solar eclipse in the fourth clime, we stated in the chapter on parallax that the [lunar] parallax cannot be greater than 64 minutes. And half the two diameters, i.e., the diameters of the Sun and Moon, do not become greater than 34 minutes. When the Moon's latitude is northerly, the latitudinal parallax must be subtracted from the lunar latitude, the remainder thus being the apparent latitude. Now at 98 minutes of latitude, whenever the latitude is northerly and the parallax is at its full maximum, there will result an apparent latitude of 34 minutes. For this reason, the maximum solar eclipse limit in the north is where the latitude is 98 minutes; there, the distance of the Moon from the node is 18 degrees plus a fraction. But since for south [latitudes] the parallax has to be added to the latitude in order to get the apparent latitude, the maximum southerly latitude in which a solar eclipse is possible is 34 minutes. Where the latitude is 34 minutes, the distance from the node is $6+\frac{1}{2}$ degrees. Then the parecliptic orb may be divided into 4 parts, similar to what we stated for the Moon. These parts, however, are unequal. The two parts that are the solar eclipse limits are each 25 degrees each; of that total, $18+\frac{1}{2}$ is in the northern direction and $6+\frac{1}{2}$ is in the southern direction. The two parts in which a solar eclipse is not possible are also unequal: the northern part is 143 degrees, and the southern part is 167 degrees, as in this illustration:



[Figure 5]

[11] And since each of these two parts, which is the solar eclipse limit, is no more than 25 degrees, a solar eclipse is not possible in [each of] two consecutive conjunctions, because the Sun will have traversed about 29 degrees during this period and be outside the solar eclipse limit. Since one part in which a solar eclipse is not possible is 143 degrees, and the Sun in five months can traverse 150 degrees, then in five months two solar eclipses are easily possible. That is so because it had passed the head in the first solar eclipse, and it had not reached the tail in the second solar eclipse. Likewise, for seven months, on the assumption that in the first solar eclipse it has not reached the tail and its distance from the tail is about 18 degrees, and seven months later that it has traversed 205 degrees, i.e., the Sun is at its slow speed, it would have passed beyond the head and would have overtaken its initial position by 7 degrees if the head had no movement. However, since the head has also moved 11 degrees, the distance is 18 degrees, and the Sun will not be outside the solar eclipse limit.

[12] Because in a southern locality the same is applicable, but south [of the ecliptic], then a solar eclipse will be possible at a distance of 18 degrees from the head in the sequence [of the signs] for northern

localities, and for southern localities, at a distance of 18 degrees in the counter-sequence, also from the head. Considering this, the solar eclipse limit will be 36 degrees, and for two successive conjunctions a solar eclipse is possible but on two [different] sides of the world, i.e., north and south; in one location, however, it is not possible.

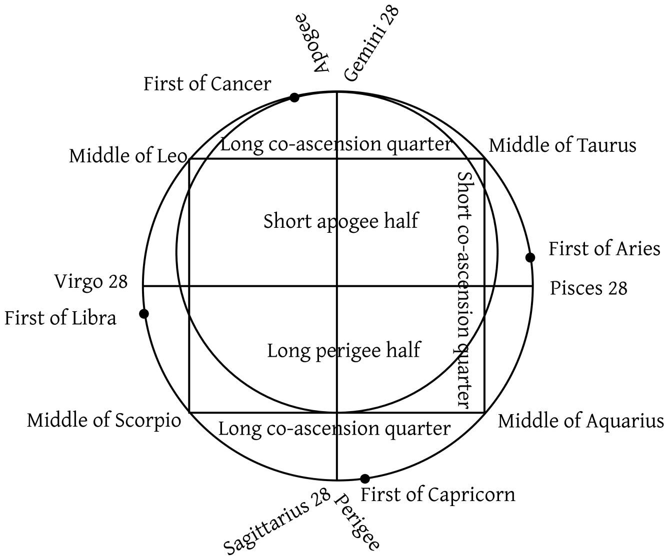
[13] This is an explanation of this matter as appropriate for this place. These values that we gave in the examples, if divergent from true ones, should be excused, inasmuch as this is not the place for correcting practical [values/calculations]. For providing an account of the configuration (*hayʾat*), this is of sufficient extent.

Section [Eight]
On Conceptualizing the Equation of Time

[1] In the Ninth Chapter of Book III, it has been stated that the difference between mean days and true days, which is expressed as the equation of time, is composed of two differences. One of these is due to fastness and slowness of the Sun on account of its uniform motion about the center of the eccentric and the consequent irregularity about the center of the World. Its greatest extent amounts to twice the maximum equation. In the apogean half, i.e., in the half of the revolution in which the apogee is at the midpoint of that half, the true days will be shorter than the mean ones because of the slower speed, while in the other half they will be longer. The second [difference] is what occurs due to the difference between degrees of the zodiacal orb and the parts on the equinoctial upon the transit of the meridian circle, or[1] rising for horizons of the equator. The maximum of this difference is 5 degrees. In the two quarters in which the two equinox points are the midpoints, i.e., from the midpoint of Aquarius to the middle of Taurus

1. Reading *yā* instead of *bā*.

and from the middle of Leo to the middle of Scorpio, the true days are shorter than the mean days; in the other two quarters longer. So, in order to combine these two differences with one another, a figure has been drawn so that it becomes clear where the two causes of shortening [the days] come together, where the two causes of lengthening [the days come together], and where these two causes are in opposition to one another. This is the figure:



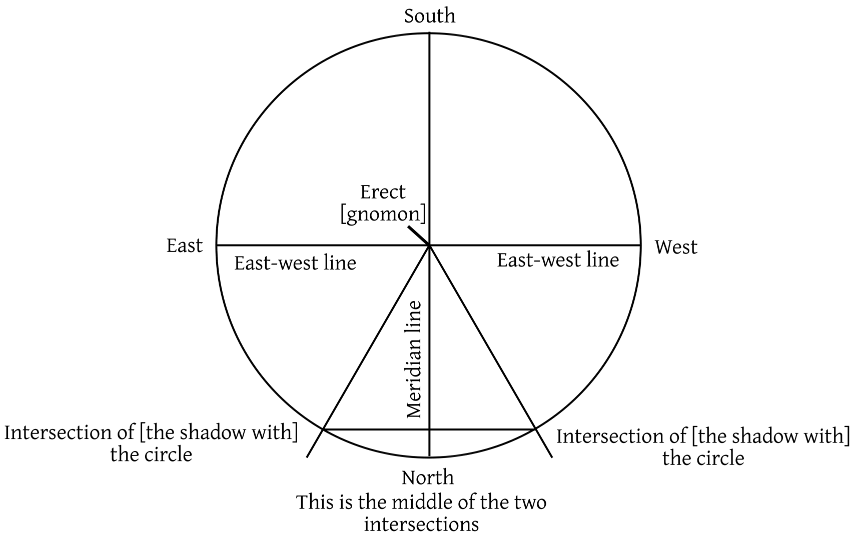
[Figure 1]

[2] According to this figure, the Sun's circuit has become divided into six parts, which are governed differently. The first part is from the midpoint of Aquarius up to Pisces 28°, approximately 43 degrees. Its days, with regard to being in the perigean half, will be long, but with regard to co-ascensions short. The second part is from Pisces 28° up to the midpoint of Taurus, approximately 47 degrees. Its days, with respect to both factors, are short. The third part, a full quarter revolution, is from the midpoint of Taurus to the midpoint of Leo. Its days, with regard to [the Sun] being in the apogean half are short, but long with regard to co-ascensions. The fourth part is from the mid-

point of Leo to Virgo 28°, approximately 43 degrees. Its days, from both points of view, are short. The fifth part is from Virgo 28° to the midpoint of Scorpio, 47 degrees. Its days are short perigean-wise, long co-ascension-wise. The sixth part is from the midpoint of Scorpio to the midpoint of Aquarius, one full quarter. Its days, from both points of view, are long. In no place have these two causes come together more than in this quarter. For this reason, it is best to make the beginning part or ending part of this quarter the starting point, so the difference will always be additive or subtractive. If the starting point is taken to be at the beginning part of [this] quarter, the true [days] will always be in excess of the mean [days]. If the starting point is taken to be at the ending part of [this] quarter, the mean [days] will always be in excess of the true [days]. This becomes quite clear in practice.

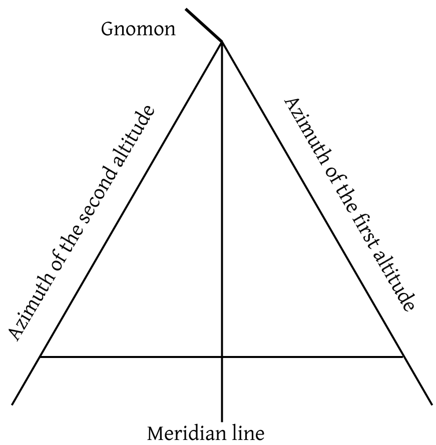
Section [Nine]
On Depicting the Indian Circle, the Azimuth of Cities, and Other Matters

[1] In Chapter Twelve of Book III, two methods have been presented to determine the meridian line. One is the Indian circle whose illustration is thus:



[Figure 1]

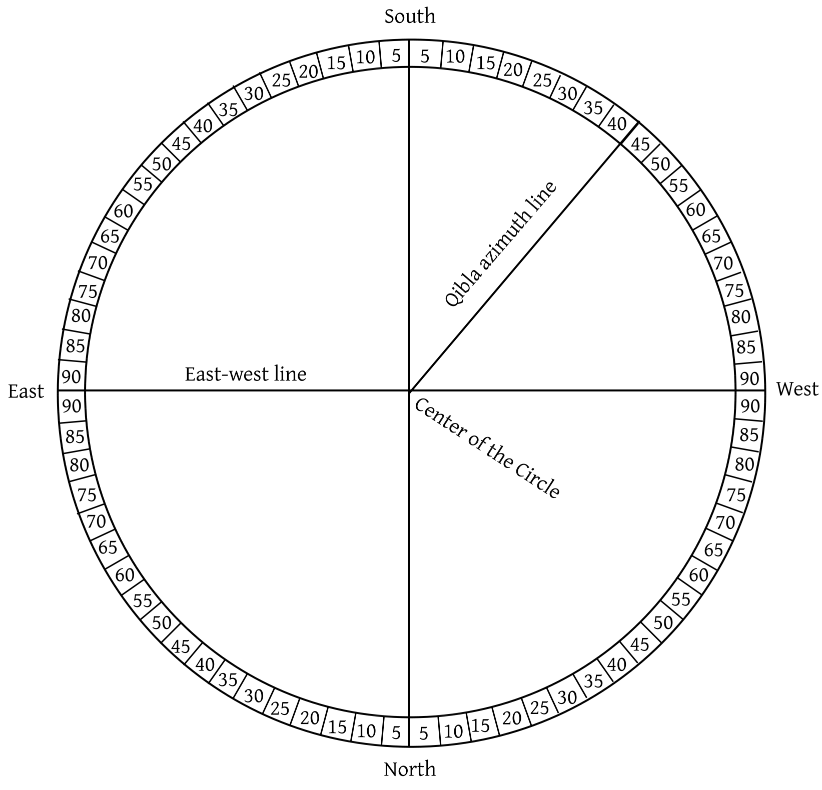
[2] The second method, by means of which the meridian line is found by the observation of two altitudes without positing a circle, is according to this illustration:



[Figure 2]

[3] When the meridian line is obtained, with the east-west line falling at right angles to it, and one marks the center of a circle at the point of intersection, and the horizon circle is drawn about it, resembling the Indian circle, then a figure will appear from [whose] circle one may

obtain the azimuth of cities. [This will be] in the manner that the circle is divided into four parts, each quarter into ninety parts. [In the amount of] the value of the azimuth of the city whose azimuth we wish to know—which can be found in practical handbooks—we move [on the circle] in the direction [of that city] and draw a line from the center to this part, so that the city's azimuth is designated. As an example, we have taken the azimuth of Mecca to be 40 degrees in the [following] illustration, and we have drawn the azimuth line [in that amount] from the south toward the west, as is clear in the figure:



[Figure 3]

Glossary

آ

آب
water

آبا
fathers

آتش
fire

آثار علوی
upper phenomena

آسمان
the heavens

آفاق مایله
oblique horizons

آفاق مستویه
straight horizons

آفتاب
Sun

آلات رصد
observational instruments

ا

ابدی‌الخفا
permanently invisible

ابدی‌الظهور
permanently visible

ابعاد و اجرام
distances and sizes

اجتماع
conjunction

اجرام بسیطه
simple bodies

اجسام
bodies

اجسام مختلفةالطبایع والصُوَر
bodies of different natures and forms

احتراق
combust

احساس
perception

اختلاف
anomaly

اختلاف منظر
parallax

ادوار دوازده‌گانه
twelve cycles

ارتفاع
altitude

اَرنَب
Lepus

استتار
concealment

استدارت
circularity

استقبال
opposition

اَسَد
Leo

اُسطُقسّات
substances

اسطوانهٔ مستدیر
circular cylinder

اسکندریه
Alexandria

اشتداد
intensifying

اشخاص
individuation

اصبع، ج اصابع
digits

اصطلاح
convention

اصول
bases

اضافت
addition

اعتبار، ج اعتبارات
consideration

اعتبار کردن
take into account

اعتدال
temperateness

اعتدال خریفی
autumnal equinox

اعتدال ربیعی
vernal equinox

اعدل مواضع
the most temperate of all localities

افق
horizon

افق، ج آفاق
locality

اقامت
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آن قرار دهیم.

در ترجمهٔ معینیه و حل ما روندی مشابه مورد تذکرهٔ طوسی و نهایة الإدراك قطب‌الدین شیرازی پی گرفتیم. [۱] سعی کردیم تا یادداشت‌هایمان ذیل ترجمه در این مجلد مختصر باشد و امیدواریم تا در مجلد سوم به شرح تفصیلی متن معینیه و حل بپردازیم.

در پایان مایلم مراتب قدردانی خود را از «بنیاد ملی علوم آمریکا» [۲] ابراز کنم که با حمایتش طی سال‌های ۲۰۰۰-۲۰۰۱م به من اجازه داد تا روی ترجمه معینیه و حل کار کنم. همچنین ما سپاسگزار خانم سالی رجب هستیم که با ویرایش و اصلاحاتش بسیار به بهبود ترجمهٔ حاضر کمک کرد.

جمیل رجب

فروردین ۱۴۰۱/آوریل ۲۰۲۲م

شیکاگو، ایلینوی آمریکا

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۱. متن تذکرهٔ خواجه نصیر توسط جمیل رجب تصحیح و به انگلیسی ترجمه شده است. مقالهٔ سوم نهایة الإدراك شیرازی نیز توسط فاطمه سوادی تصحیح و ترجمه شده است:

F. J. Ragep, *Naṣīr al-Dīn al-Ṭūsī's Memoir on Astronomy (al-Tadhkira fī ʿilm al-hayʾa),* 2 vols. (New York: Springer-Verlag, 1993). Fateme Savadi, "*The Historical and Cosmographical Context of* Hayʾat al-arḍ *with a focus on Quṭb al-Dīn Shīrāzī's* Nihāyat al-Idrāk" (PhD diss., McGill University, 2019).

۲.

US National Science Foundation (Grant no. SES9911005)

از نسخه‌های دیگر برخی از این خطاها اصلاح شده است. لیکن به نظر می‌رسد این اصلاحات ثانوی توسط کسانی غیر از طوسی انجام شده است. مورد دیگری که حتی مسألهٔ را پیچیده‌تر می‌کند این است که خود طوسی در تحریر المجسطي که زمانی کوتاه پس از نگارش حل، در ۶۴۴ق نوشته شده است، اعداد درست مربوط به صورت‌های فلکی را ارائه داده است، یعنی اعداد تحریر با آن‌چه در خود مجسطی بطلمیوس یافته می‌شود تطابق دارد. در این موارد نیز تصمیم بر این بوده است تا آن‌چه در نسخه‌های معتبر آمده، ترجمه شود و از این نظر تصحیح و ترجمه با هم تطابق دارند.

مسألهٔ مشابهی در مورد تصاویر وجود دارد. در این موارد نیز این وسوسه وجود داشته تا اشتباهاتی که در تصاویر وجود دارد در متن مصحح و ترجمه «اصلاح» شود. اما چنین کاری مثل تصحیح یوهان لودویگ هایبرگ [1] از متون ریاضی یونانی، به تحریف متن و ارائهٔ تصویری نادرست از تاریخ آن منجر می‌شود. [2] تصاویر معینیه و حل عموماً بر اساس نسخهٔ «ج» و با آگاهی از خطاهای احتمالی خود طوسی یا کاتبان نسخه‌ها ترسیم شده‌اند (یک نمونه تصویر شمارهٔ ۵ در فصل هفتم حل است). برای کمک به خواننده در مقایسهٔ تصاویر ما با آن‌چه در نسخه‌های خطی اثر طوسی یافته می‌شود، ما قصد داریم تا پس از انتشار این مجلد این تصاویر را به صورت آنلاین در کنار متن مصحح و ترجمهٔ

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۱. Johan Ludvig Heiberg

۲. در این مورد به منابع زیر مراجعه کنید:

Ken Saito and Nathan Sidoli, "Diagrams and Arguments in Ancient Greek Mathematics: Lessons Drawn from Comparisons of the Manuscript Diagrams with Those in Modern Critical Editions," 135-62 and Reviel Netz, "The Texture of Archimedes' Writings: Through Heiberg's Veil," 163-205, in *The History of Mathematical Proof in Ancient Traditions*, ed. Karine Chemla (Cambridge: Cambridge University Press, 2012).

سیاره‌ای ابتکاری‌اش بر مبنای «جفت طوسی» را شناسایی کنیم. دوم این‌که این روایت اطلاعات ارزشمندی دربارهٔ حامیان طوسی در دربار اسماعیلی، و تأثیر این مسأله بر روش ارائهٔ مطالبش در اختیار می‌گزارد. سوم این‌که با وجود یکسان بودن بیش‌تر بخش‌های روایت اصلاح شدهٔ متن طوسی با روایت اولیه (به استثنای مقدمه‌ها)، اصلاحات انجام شده روی متن نمایانگر گوشه‌هایی از تاریخ متن است. برخی از این اصلاحات به خود طوسی برمی‌گردد، برخی احتمالاً توسط دیگران انجام گرفته است. نتیجه‌ای که می‌توانیم از وجود این اصلاحات بگیریم این است که طوسی مرتکب «اشتباهاتی» در پیش‌نویس متن خود شده بوده است. منظور از این اشتباهات خطاهای املایی، دستور زبانی، عددی، و مفهومی آشکاری است که در برخی موارد توسط خود طوسی، شاگردانش، کاتبان نسخه‌های خطی، یا دیگران برطرف شده است. از آن‌جاکه بهترین نسخهٔ نمایندهٔ روایت اسماعیلی معینیه یعنی نسخهٔ شمارهٔ ۱۳۴۶ کتابخانهٔ مرکزی دانشگاه تهران (نسخهٔ «ج»)، دربرگیرندهٔ اصلاحات بعدی طوسی نیز است، می‌توان نتیجه گرفت که خواجه به خاطر تعجیل یا به هر دلیل دیگری، دقتی را که در زمان تألیف التذکرة في علم الهیئة داشت، در اصلاح متن معینیه نشان نداد.

این مسأله برای مترجمی که با اشتباهاتی آشکاری مانند عدد ۳۳۲۴۶۰۹۵۲ به جای عدد صحیح ۳۳۴۶۰۹۵۲ مواجه می‌شود (نک : معینیه، مقالهٔ چهارم، باب پنجم، بند هفتم) یک چالش جدی محسوب می‌شود. چنین اشتباهاتی را می‌توان خطای کتابت در نظر گرفت. ولی از آن‌جا که نسخه‌های نمایندهٔ روایت اسماعیلی دربرگیرندهٔ این اشتباهات هستند، تصمیم گرفتیم که آن‌ها را در متن تصحیح شده و در ترجمه حفظ کنیم و شکل اصلاح شده را در پاورقی بیاوریم. بر اساس شواهد موجود این اشتباه و موارد مشابه دیگر از خود طوسی نشأت گرفته بوده است. مورد دیگر از این اشتباهات در اعداد فهرست صورت‌های فلکی دیده می‌شود. بهترین نسخهٔ ما (و نیز بیشتر نسخه‌های به کار رفته در تصحیح) در بردارنده‌ی این اعداد نادرست هستند، در حالی‌که در تعدادی

متأسفانه تعهدات دیگر مدام مانع می‌شد و سال‌های بسیاری گذشت و این پروژه به پایان نرسید. تا این‌که فاطمه سوادی و سجاد نیک‌فهم خوب‌روان زمانی که دانشجوی دکتری دانشگاه مک‌گیل بودند متن این دو اثر خواجه را تصحیح کردند. ترجمهٔ اولیهٔ ثکستون در به سرانجام رساندن ترجمهٔ حاضر ما را خیلی یاری رساند، اگرچه معلوم شد اصلاحات بسیاری لازم دارد. امیدواریم از این طریق بتوانیم مراتب سپاس‌گزاری خود را از ثکستون به خاطر این‌که اجازه داد تا از ترجمهٔ اولیه‌اش بهره ببریم به جا بیاوریم و نقش او را در آن‌چه در این مجلد عرضه می‌شود به اطلاع خوانندگان برسانیم.

قبل از پرداختن به ترجمهٔ حاضر لازم است منظورمان از تصحیح انتقادی متن معینیه و حل را به اختصار توضیح دهیم (برای توضیحات تفصیلی لطفاً به مقدمهٔ جلد اول کتاب مراجعه نمایید). طوسی اولین روایت معینیه را در ۶۳۲ق به پایان رساند. حل که در برخی از نسخه‌های خطی با نام ذیل نیز شناخته می‌شود در ۶۴۳ق به عنوان ضمیمه به متن اصلی افزوده شد. ولی این پایان داستان نبود. خواجه هم‌چنان به انجام اصلاحاتی در متن ادامه داد که مهم‌ترین آن‌ها تغییراتی است که در مقدمهٔ هر دو اثر اعمال کرد و به واسطهٔ آن هرگونه ذکر حامیان اولیه خود یعنی حاکمان اسماعیلی قهستان در شرق ایران را حذف کرد. این رخداد بی‌تردید پس از آن روی داد که ایران تحت حکومت ایلخانان قرار گرفت و طوسی به خدمت آنان درآمد. سوادی و نیک‌فهم از طریق بررسی‌های نسخه‌شناسی دامنه‌دار توانستند تصحیحی انتقادی از آن‌چه که کهن‌ترین روایت اسماعیلی متن این دو اثر می‌نامیم در جلد اول کتاب حاضر ارائه کنند. این متن نزدیک‌ترین روایت به متن اولیهٔ طوسی است که می‌توان فعلاً به دست داد. سازوارهٔ انتقادی به خواننده امکان بازسازی روایت‌های بعدی را می‌دهد.

اساس قرار دادن روایت اولیهٔ اسماعیلی، به جای روایت اصلاح شده که در آن نشانه‌های اسماعیلی زدوده شده، برای تصحیح متن بنا به چند دلیل انجام شد. اول این‌که چنین متنی به ما این امکان را می‌دهد که نظریه‌های اولیهٔ نجومی طوسی و مدل‌های

پیش‌گفتار

مایهٔ افتخار است که در این مجلد ترجمهٔ انگلیسی الرسالة المعینیة (الرسالة المغنیة) و ذیل آن، حل مشکلات معینیه، نوشتهٔ خواجه نصیر طوسی را که تصحیح آن‌ها در مجلد اول این کتاب منتشر شد، در اختیار خوانندگان قرار دهیم.[۱] ترجمه‌ای که صرف بیان به درازا کشیده شدن فرآیند آماده‌سازی‌اش به هیچ عنوان حق مطلب را ادا نمی‌کند.

من کار ترجمهٔ این دو متن را با همکاری ویلر ثکستون، استاد دانشگاه هاروارد، در دههٔ هشتاد میلادی آغاز کردم. در آن زمان ما بیشتر روی حل کار کردیم و ترجمهٔ اولیه‌ای از متن فراهم آوردیم. اما در کمال شگفتی، البته از نوع خوشایند آن، ثکستون در سال ۱۹۹۲م ترجمه‌ای کامل از معینیه به همراه ترجمهٔ بازبینی‌شدهٔ حل را برایم فرستاد.

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۱. نصیرالدین محمد طوسی، الرسالة المعینیة (الرسالة المغنیة) و حل مشکلات معینیه، جلد اول: متن مصحح فارسی، تحقیق: سجاد نیک‌فهم خوب‌روان و فاطمه سوادی، با دو پیشگفتار از جمیل رجب و حسین معصومی همدانی، تهران، میراث مکتوب، ۱۳۹۹ش.

الرسالة المعينية (الرسالة المغنية)

و

حلّ مشکلات معینیه

جلد دوم

تألیف: نصیر الدین محمد طوسی

ترجمهٔ انگلیسی: جمیل رجب ـ فاطمه سوادی ـ سجاد نیک‌فهم خوب‌روان

ناشر: میراث مکتوب

مدیر تولید: محمد باهر

خطاطی عنوان روی جلد: احمد عبدالرضایی

مدیر فنّی و امور چاپ : حسین شاملوفرد

چاپ اوّل : ۱۴۰۲

شمارگان : ۱۰۰ نسخه

بها: ۲۳۵۰۰۰۰ تومان

شابک (دوره): ۱ ـ ۲۰۴ ـ ۲۰۳ ـ ۶۰۰ ـ ۹۷۸

شابک (ج ۲): ۱ ـ ۲۸۸ ـ ۲۰۳ ـ ۶۰۰ ـ ۹۷۸

چاپ (دیجیتال): میراث

نشانی ناشر : تهران، ش. پ: ۱۳۱۵۶۹۳۵۱۹

تلفن: ۶۶۴۹۰۶۱۲، دورنگار: ۶۶۴۰۶۲۵۸

E-mail: tolid@MirasMaktoob.ir

http://www.MirasMaktoob.ir

این کتاب با کاغذ حمایتی منتشر شده است

نصیر الدّین محمّد طوسی

الرِّسالَةُ المُعینیّة

(الرِّسالَةُ المُغنیَة)

وحلّ مشکلاتِ مُعینیّه

جلد دوم

ترجمهٔ انگلیسی

جمیل رجب

فاطمه سوادی

سجاد نیک فهم خوب روان

میراث مکتوب

۳۴۶

علوم و فنون

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متون و پژوهشهایی در تاریخ علم

(۲)

دبیر مجموعه

حسین معصومی همدانی

سرشناسه : نصیرالدین طوسی، محمدبن محمد، ۵۹۷ ـ ۶۷۲ق.
Nasir al-Din Tusi, Muhammad ibn Muhammad
عنوان و نام پدیدآور : الرسالة المعینیة (الرسالة المغنیة) و حل مشکلات معینیه / نصیرالدین محمد طوسی؛ تصحیح و تحقیق سجاد نیک فهم خوب‌روان، فاطمه سوادی؛ با دو پیش‌گفتار از جمیل رجب، حسین معصومی همدانی؛ با همکاری پژوهشکده تاریخ علم دانشگاه تهران.
مشخصات نشر : تهران: مؤسسهٔ پژوهشی میراث مکتوب، ۱۳۹۹ ـ
مشخصات ظاهری : ۲ ج.: نمونه (رنگی).
فروست : میراث مکتوب: ۳۴۶. علوم و فنون؛ ۲۵
شابک : دوره: 978-600-203-204-1 ج. ۱: 978-600-203-203-4 ج. ۲: 978-600-203-288-1
وضعیت فهرست‌نویسی: فیپا
یادداشت : ج.۲ (چاپ اول: ۱۴۰۲) (فیپا)
یادداشت : کتابنامه.
یادداشت : نمایه.
مندرجات : ج.۱. متن مصحح فارسی
موضوع : نجوم ـ متون قدیمی تا قرن ۱۴
موضوع : Astronomy-Early work to 20th century
موضوع : نجوم اسلامی ـ متون قدیمی تا قرن ۱۴
موضوع : Islamic astronomy-Early works to 20th century*
شناسه افزوده : نیک‌فهم خوب‌روان، سجاد، ۱۳۶۱ ـ
شناسه افزوده : سوادی، فاطمه، ۱۳۵۹ ـ
شناسه افزوده : رجب، جمیل، ۱۹۵۰ ـ ، مقدمه‌نویس
شناسه افزوده : معصومی همدانی، حسین، ۱۳۲۷ ـ ، مقدمه‌نویس
شناسه افزوده : مؤسسهٔ پژوهشی میراث مکتوب
شناسه افزوده : دانشگاه تهران. پژوهشکده تاریخ علم
رده‌بندی کنگره : QB ۴۱
رده‌بندی دیویی : ۵۲۰
شماره کتابشناسی ملی : ۷۲۹۶۷۵۳
اطلاعات رکورد کتابشناسی : فاپا

بسم الله الرحمن الرحیم